CSASC 2011 - Minisymposium on “Complex Analysis”, Sept. 25 – 28, 2011, Krems

Organizers: Franc Forstnerič (Ljubljana), Martin Kolář (Brno), Bernhard Lamel (Wien)

Time Schedule

Sunday	**Audi-Max**
13:45 - 14:45	Franc Forstnerič (Plenary talk)
14:45 - 15:15	*Coffee Break*
15:15 - 16:00	Jasna Prezelj
16:00 - 16:45	Laurent Stolovitch
16:45 - 17:30	Dmitri Zaitsev
17:30 - 17:50	Giuseppe della Sala
17:50 - 18:10	Anne-Katrin Herbig

Monday	**Seminar room SE C 2.08**
10:30 - 11:15	Jean Ruppenthal
11:15 - 12:00	Friedrich Haslinger
12:00 - 12:30	Elizabeth Wulcan

Tuesday	**Seminar room SE C 2.08**
14:45 - 15:15	Erlend Fornæss Wold
15:15 - 16:00	Marko Slapar
16:00 - 16:45	Francine Meylan

Wednesday	**Seminar room SE C 2.08**
10:30 - 11:15	Barbara Drinovec Drnovšek
11:15 - 11:35	Joe J. Perez
11:35 - 11:55	Florian Bertrand

Abstracts

Florian Bertrand: Common boundary values of holomorphic functions for two-sided complex structures

We consider two disjoint domains in $\mathbb{C}^n$, whose boundaries share a smooth real hypersurface M as relatively open subsets. Each domain is endowed with a complex structure which is smooth up to M. And we study the smoothness up to M of a continuous function which is holomorphic on each domain with respect to the structures. Although the result we prove is more meaningful for integrable structures, our methods make it much more natural to deal with the general almost complex structures. This is a joint work with Xianghong Gong and Jean-Pierre Rosay.

Barbara Drinovec Drnovšek: The Poletsky-Rosay theorem on singular complex spaces

If u is an upper semicontinuous function on a locally irreducible complex space X, then the largest plurisubharmonic function v that is less than or equal to u is obtained as the pointwise infimum of the averages of u over the boundaries of analytic discs in X. This was proved by Poletsky (1993) for $X = \mathbb{C}^n$ and by Rosay (2003) for X a complex manifold. Applications include the description of the plurisubharmonic hull of a compact set in a complex space.

Giuseppe della Sala: Watson's Lemma and the flatness of CR functions

For a certain class of real hypersurfaces of $\mathbb{C}^n$, we study "how fast" can a CR function defined on them vanish without being trivial. As it turns out, the possible rate of vanishing depends on certain results about holomorphic functions in one complex variable.

Franc Forstnerič: What is an Oka manifold?

Two of the classical theorems in the theory of holomorphic functions are the Runge approximation theorem and the Weierstrass interpolation theorem. A complex manifold Y is said to be an Oka manifold if these results, and some of their natural extensions, are valid for holomorphic maps from any Stein manifold (in particular, from complex Euclidean spaces) to Y. After a brief review of the development of this subject, beginning with the classical Oka-Grauert theory and continuing with the seminal work of Gromov, I will describe some of the recent developments and future challenges in this field of complex geometry.

Friedrich Haslinger: Compactness of the $\bar{\partial}$- Neumann operator and applications to Schrödinger and Dirac operators

We use a description of precompact subsets in L^2-spaces to characterize compactness of the $\bar{\partial}$-Neumann operator

$$N : L^2_{(0,q)}(\Omega) \to L^2_{(0,q)}(\Omega),$$

where $\Omega \subset \mathbb{C}^n$ is a bounded pseudoconvex domain, or

$$N_\varphi : L^2_{(0,q)}(\Omega, e^{-\varphi}) \to L^2_{(0,q)}(\Omega, e^{-\varphi}),$$

where $\Omega \subseteq \mathbb{C}^n$ is a pseudoconvex domain and φ is a plurisubharmonic weight function. It turns out that Gårding's inequality in the interior and and the boundary behavior (respectively the behavior at infinity) of suitable plurisubharmonic functions (respectively of the weight function) play a crucial role. Using the general characterization of compactness it is easy to show that property (P) implies compactness of N, if Ω is a smoothly bounded pseudoconvex domain, and that a certain behavior at infinity of the eigenvalues of the Levi matrix of the weight function φ imply compactness of N_φ.
In the case $L^2(\mathbb{C}^n, e^{-\varphi})$ there is a relationship to Schrödinger operators with magnetic field and Pauli and Dirac operators and to the complex Witten Laplacian. We show that the above results on the $\bar{\partial}$-Neumann operator can be used to settle the question whether these operators have compact resolvent. In this connection it is important to know whether the Fock space

$$\mathcal{A}^2(\mathbb{C}^n, e^{-\varphi}) = \{f : \mathbb{C}^n \to \mathbb{C} \text{ entire} : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi} d\lambda < \infty\}$$

is infinite-dimensional, which again depends on the behavior at infinity of the eigenvalues of the Levi matrix of the weight function φ.

F. Haslinger, Compactness for the $\bar{\partial}$- Neumann problem - a functional analysis approach, ESI-preprint 2208, arXiv:0912.4406 , Collectanea Mathematica 62 (2011), 121-129.
F. Haslinger, Compactness of the $\bar{\partial}$- Neumann operator on weighted $(0, q)$-forms. ESI preprint 2291, arXiv: 1012.433 , Proceedings of the IWOTA Conference 2010, Birkhäuser Verlag, to appear.

Anne-Katrin Herbig: Smoothing properties of the Bergman projection

Let D be a bounded domain in $\mathbb{C}^n$ with smooth boundary. If the Bergman projection, B, on D is sufficiently regular, then it actually exhibits certain smoothing behaviors. E.g., if B is exactly regular, then derivatives of the output of B, measured in $L^2(D)$, only depend on derivatives of the input in a certain tangential direction.
This is joint work with Jeff McNeal and Emil Straube.

Francine Meylan: Local models for strictly pseudoconvex CR structures of hypersurface type

In this talk, we will analyse all the strictly pseudoconvex CR structures of hypersurface type on the standard contact structure on $\mathbb{R}^{2n+1}$.

Joe J. Perez: Unitary representations of unimodular Lie groups in Bergman spaces

For an arbitrary unimodular Lie group G, we construct strongly continuous unitary representations in the Bergman space of a strongly pseudoconvex neighborhood of G in the complexification of its underlying manifold. These representation spaces are infinite-dimensional and have compact kernels. In particular, the Bergman spaces of these natural manifolds are infinite-dimensional.

Jasna Prezelj: Relative Oka-Grauert principle on 1-convex spaces

We present the relative Oka-Grauert principle for holomorphic submersions over 1-convex spaces using conic neighbourhoods of holomorphic sections over 1-convex spaces.
As an application we have the generalized Oka-Grauert principle for 1-convex manifolds: Every continuous mapping X to Y from a 1-convex manifold X to a complex manifold Y which is already holomorphic on a neighborhood of the exceptional set is homotopic to a holomorphic one provided that either Y satisfies the CAP property or we are free to change the complex structure on X.

Jean Ruppenthal: L^2-cohomology of complex spaces with isolated singularities

Let X be a compact complex space with isolated singularities. In this talk, we give a complete description of the L^2 Dolbeault cohomology for $(0, q)$-forms on X in terms of a resolution of singularities $\pi : M \to X$. In fact, the L^2 cohomology of X is equivalent to the cohomology of M with values in a certain holomorphic line bundle modulo some torsion which sits on the exceptional set of the desingularization. The idea how this torsion can be described for all kinds of isolated singularities goes back to an observation made by Øvrelid and Vassiliadou earlier this year.

Marko Slapar: On CR singular points of real manifolds in complex manifolds

Let M be a real manifold in a complex manifold X. The dimension of a maximal complex subspace T_p point of a generic point $p \in M$ equals $m - n$, with $m \geq n$ respectively being real and complex dimension of M and X. The points where this dimension is grater than $m - n$ are called CR singular. We will review the situation $m = n = 2$ and give some new results in the case $m = 4, n = 3$.

Laurent Stolovitch: Holomorphic transformations to normal forms in sectorial domains

We will talk about local classification of germs of holomorphic vector fields vanishing at 0. It is always possible to transform a vector field into a simpler model, called a normal form. Such a transformation is usually a divergent formal power series at the origin. Nevertheless, we will show that some germs of holomorphic vector fields can be transformed into a polynomial normal form by mean of an holomorphic map defined in a sectorial domain which contains 0 in the boundary.

Erlend Fornæss Wold: The Hartog's extension phenomenon for line bundles across totally real manifolds

Let X be a complex manifold of dimension greater than or equal to three, and let M be a closed real analytic totally real submanifold of X. Then any holomorphic line bundle over X/M extends to X. The most interesting case is when $\dim X = \dim M = 3$, in which case there are locally near points in M topologically nontrivial line bundles. This is joint work with Fornæss and Sibony.

Elizabeth Wulcan: Effective membership problems on varieties

This talk is based on a joint work with Mats Andersson, in which we use residue currents to bound the degrees of solutions to polynomial ideal membership problems on algebraic varieties. In particular, I will discuss a generalization to the non-smooth case of a global Briancon-Skoda theorem due to Hickel and Ein-Lazarsfeld.

Dmitri Zaitsev: Dynamics of one- and multi-resonant biholomorphisms

We construct a simple formal normal form for holomorphic diffeomorphisms in $\mathbb{C}^n$ whose differentials have one-dimensional family of resonances in the first m eigenvalues, $m \leq n$ (but more resonances are allowed for other eigenvalues). Next, we provide invariants and give conditions for the existence of basins of attraction and extend them to the case of larger sets of resonances. Finally, we give applications and examples demonstrating the sharpness of our conditions. This is a joint work with Filippo Bracci and Jasmin Raissy.