An analysis of exponential Taylor integrators

Antti Koskela and Alexander Ostermann
University of Innsbruck

We consider a Taylor series based exponential integrator for the time integration of large stiff systems of ordinary differential equations, which result from semidiscretization of partial differential equations and which are of the form $u'(t) = Au(t)+g(u(t))$. The integrator can be obtained by Taylor series expansion of the nonlinear part $g(u(t))$ at each numerical approximation u_n, and is given by a sum of the form $\sum_{k=0}^{p} h^k \varphi_k(hA)w_k$. The matrix functions φ_k are related to the exponential function, and the coefficients w_k represent time derivatives of $g(u(t))$ at u_n. The computational attractiveness of this method comes from a result of Al-Mohy and Higham [1], which states that this sum can be expressed in terms of a single exponential of a matrix $\widetilde{A}$ built by augmenting A with p additional rows and columns. The integrator works well for small values of p. For $p \geq 4$, however, the method suffers from instabilities which are caused by the loss of smoothness in the numerical solution along the time integration. The accumulation of round-off errors is demonstrated. Moreover, we perform numerical comparisons for the case of a state-independent inhomogeneity $u'(t) = Au(t) + g(t)$, where these instabilities do not occur. For the case of a state-dependent inhomogeneity we shortly discuss the efficient computation of the Taylor coefficients using the principles of automatic differentiation. Numerical experiments supporting the theoretical analysis are given using MATLAB.

References

[1] Awad H. Al-Mohy and Nicholas J. Higham: Computing the action of the matrix exponential, with an application to exponential integrators, SIAM J. Sci. Comput. 33 (2011) pp. 488–511.

[2] Antti Koskela and Alexander Ostermann: Exponential Taylor methods—analysis and implementation, University of Innsbruck, preprint, 2011.