Approximate solution of initial-boundary value problems with nonperiodic Fourier series

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For solving parabolic (i.e. heat) equations or hyperbolic (i.e. vibrating string) equations, the method of lines is an alternative to finite difference methods or finite elements method. This method consists of discretizing the equation in space variables and integrating the remaining initial value ODEs.

We choose truncated nonperiodic trigonometric series for the spatial discretization and a Magnus method for the integration in time. The trigonometric series approximation with its spectral convergence allows us to use a smaller ODE system, which enables us to apply the Magnus method for time integration. The approximation with nonperiodic trigonometric series is based on Huybrechts' technique for approximating nonperiodic functions by trigonometric series.

In the first part of the talk we'll describe the efficient construction of the half-range Chebyshev polynomials of the first and second kind and their role in approximating functions with trigonometric series, as well as efficient manipulations with these series (computing derivatives and products of two such series). In the second part applications of these techniques to BVPs will be considered. Finally, in the third part we will apply the Magnus method to the resulting IVP in ODEs.