THE NUMBER OF B_h-SETS AND AN ASSOCIATED EXTREMAL PROBLEM FOR RANDOM SETS OF INTEGERS

Y. KOHAYAKAWA (U. OF SÃO PAULO)

Let A be a set of integers. For any integer $h \geq 2$, we say that A is a B_h*-set* if the h-wise sums

$$a_1 + \cdots + a_h \qquad (a_1, \ldots, a_h \in A,\ a_1 \leq \cdots \leq a_h)$$

are all distinct. Let $[n] = \{1, \ldots, n\}$. It is natural to investigate the extremal function

$$F_h(n) = \max\{|A|\colon A \subset [n] \text{ is a } B_h\text{-set}\}.$$

The particular case of this problem in which $h = 2$ was raised by Simon Sidon in the 1930s and B_2-sets are known as *Sidon sets*. An immediate argument shows that $F_h(n) = o(n)$ and, therefore, this is a so called ‘degenerate’ extremal problem. As it is often the case with degenerate problems, the structure of the *extremal*, that is, largest, B_h-sets $A \subset [n]$ is not well understood.

We address the simpler problem of estimating the *number* of B_h-sets $A \subset [n]$ of a given cardinality, and obtain good bounds. We also discuss a related extremal problem: we consider random sets of integers $R \subset [n]$ of a given cardinality $m = m(n)$ and study $F_h(R)$, the maximum cardinality of a B_h-set contained in R. The behaviour of $F_h(R)$ as $m = m(n)$ varies is somewhat unexpected, presenting two points of ‘phase transition’.

This is joint work with D. Dellamonica Jr, S. J. Lee, V. Rödl and W. Samotij.