ČECH HOMOTOPY GROUPS OF ONE-DIMENSIONAL CONTINUA (CLASSIFICATION OF THE INVERSE LIMITS OF FREE GROUPS OF FINITE RANK)

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We outline proofs of the following result.

Theorem. (E-Nakamura [EN]) Let X be a one-dimensional continuum. Then, the Čech homotopy group (shape group) $\check{\pi}_1(X)$ is isomorphic to one of the following groups (1)-(5). Conversely, each of the groups (1)-(5) is isomorphic to some $\check{\pi}_1(X)$. Moreover, groups (1)-(5) are mutually non-isomorphic.

(1) free groups of finite rank;

The Čech homotopy group of the Hawaiian earring (the canonical inverse limit of free groups of finite rank).

(2) $\varprojlim(G_n, p_n : n < \omega)$ where $G_n = *_{i<n}\mathbb{Z}_i$ and $p_n : G_{n+1} \to G_n$ is the projection such that $p_n \mid *_{i<n} \mathbb{Z}_i = \mathrm{id}$ and $p_n(\mathbb{Z}_n) = \{e\}$;

(3) the free group of countable rank F_ω;

Start from F_ω and perform the construction of (2).

(4) $\varprojlim(G_n, p_n : n < \omega)$ where $G_0 = F_\omega$, $G_{n+1} = G_n * \mathbb{Z}_n$ and $p_n : G_{n+1} \to G_n$ is the projection such that $p_n \mid G_n = \mathrm{id}$ and $p_n(\mathbb{Z}_n) = \{e\}$;

Mimick the construction of (2) and use copies of F_ω instead of those of $\mathbb{Z}$.

(5) $\varprojlim(G_n, p_n : n < \omega)$ where $G_0 = F_\omega$ and $G_{n+1} = G_n * F_{\omega n}$ where $F_{\omega n}$ is a copy of F_ω, $p_n : G_{n+1} \to G_n$ is the projection such that $p_n \mid G_n = \mathrm{id}$ and $p_n(F_{\omega n}) = \{e\}$.

This is distinct from the case of the fundamental group of the Hawaiian earring, where we have free groups of finite rank as its free factors.

References

[EN] K. Eda and J. Nakamura, *The classification of the inverse limits of sequences of free groups of finite rank*, Bull. London Math. Soc., in press.