UE Discrete Mathematics

Exercises for Jan 23, 2024

131) Show that $\sqrt{2}+\sqrt{3}$ is algebraic over $\mathbb{Q}$ and determine its minimal polynomial.

132) Each element of $\mathbb{Q}(\sqrt[3]{2})$ can uniquely be represented in the form $a+b\sqrt[3]{2}+c\sqrt[3]{4}$ where $a,b,c\in\mathbb{Q}$. Why is this true?
Consequently, each element of $\mathbb{Q}(\sqrt[3]{2})$ can be identified with a triple $(a,b,c)\in\mathbb{Q}^3$. Which triple corresponds to $(a+b\sqrt[3]{2}+c\sqrt[3]{4})(a'+b'\sqrt[3]{2}+c'\sqrt[3]{4})$?

133) Which of the following polynomials is primitive over $\mathbb{Z}_3$?

$$x^3+x^2+x+1, \quad x^3+x^2+x+2, \quad x^3+2x+1.$$

134) Let K be a field with $\mathrm{char}(K)=p$. Prove that $(a+b)^p=a^p+b^p$ for all $a,b\in K$.

Hint: Use the binomial theorem and consider the equation $\binom{p}{k}=p\cdot\frac{(p-1)!}{k!(p-k)!}$ for $0<k<p$. Show that $\binom{p}{k}\in\mathbb{N}$ implies that the fraction on the right-hand side must be an integer, too, since the factors in the denominator do not divide p.

135) Four symbols have to be encoded with elements of $\mathbb{Z}_2^5$ such that the code forms a $(5,k)$ linear code (k to be determined) with which 1-bit errors can be detected and corrected. Determine a generating matrix and a check matrix of this code.