

DIJKSTRA'S ALGORITHM

Dijkstra's Algorithm

$l(v_0) := 0$; FOR $v \in V \setminus \{v_0\}$ DO: $l(v) := \infty$; END; $U := \{v_0\}$; $u := v_0$;

DO:

$C := \text{false}$;

FOR $v \in V \setminus U$ DO:

IF $(u, v) \in E$ AND $l(v) > l(u) + w(u, v)$ THEN

$p(v) := u$;

$l(v) := l(u) + w(u, v)$;

END IF;

END;

IF $\exists v \in V \setminus U : l(v) < \infty$ THEN

$C := \text{true}$;

choose node $z \in V \setminus U$ with $l(z) = \min_{v \in V \setminus U} l(v)$;

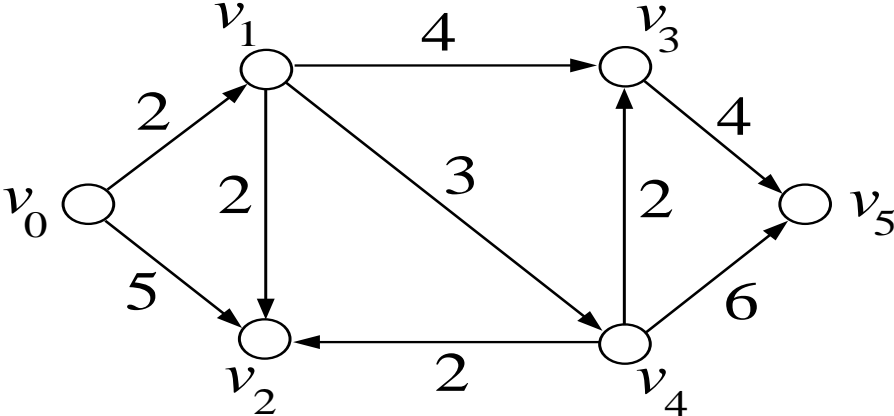
$U := U \cup \{z\}$;

$u := z$;

END IF

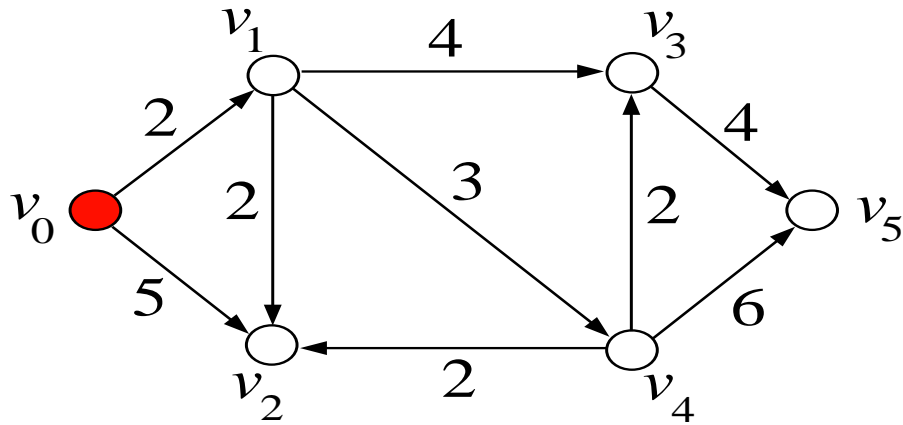
WHILE $U \neq V$ AND $C = \text{true}$;

Dijkstra's Algorithm



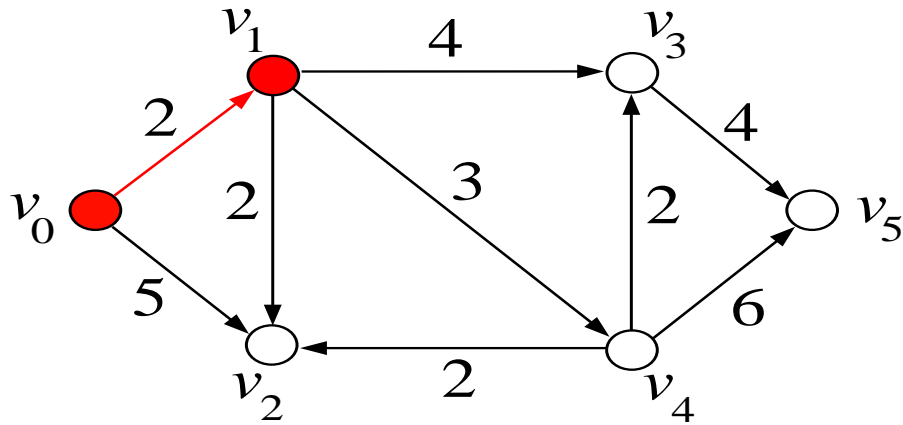
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	

Dijkstra's Algorithm



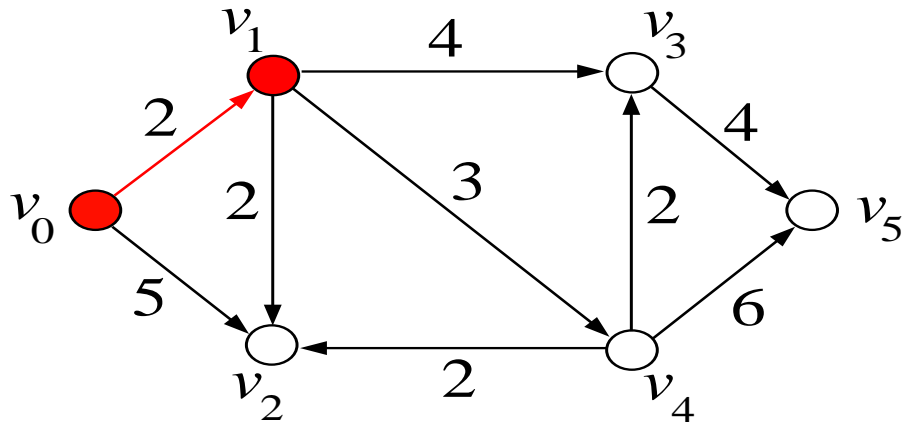
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	

Dijkstra's Algorithm



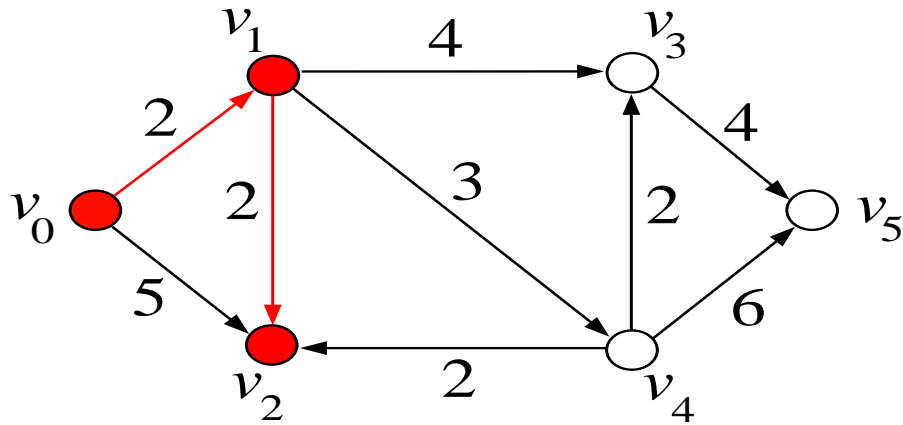
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1

Dijkstra's Algorithm



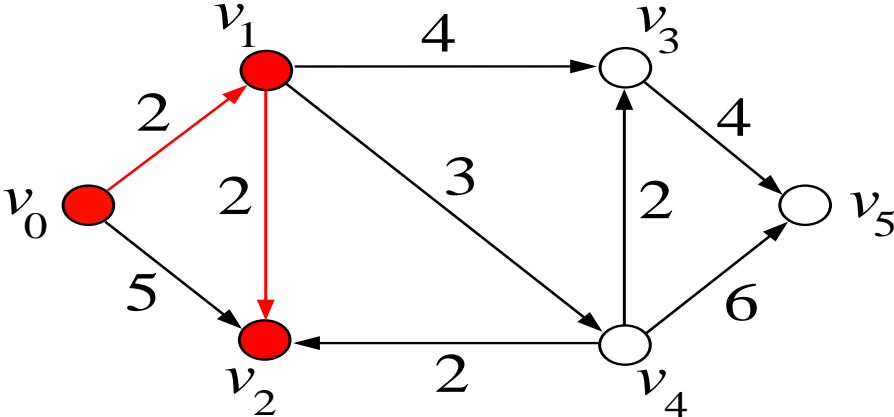
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1
2			$4/v_1$	$6/v_1$	$5/v_1$	∞	

Dijkstra's Algorithm



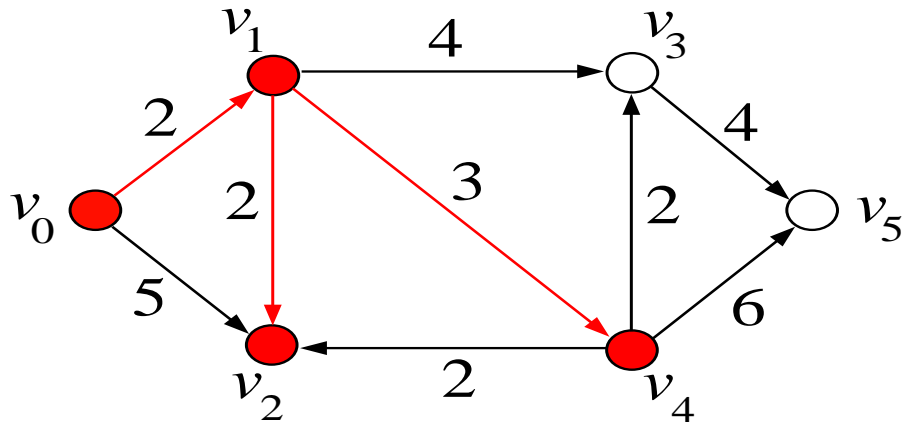
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1
2			$4/v_1$	$6/v_1$	$5/v_1$	∞	v_2

Dijkstra's Algorithm



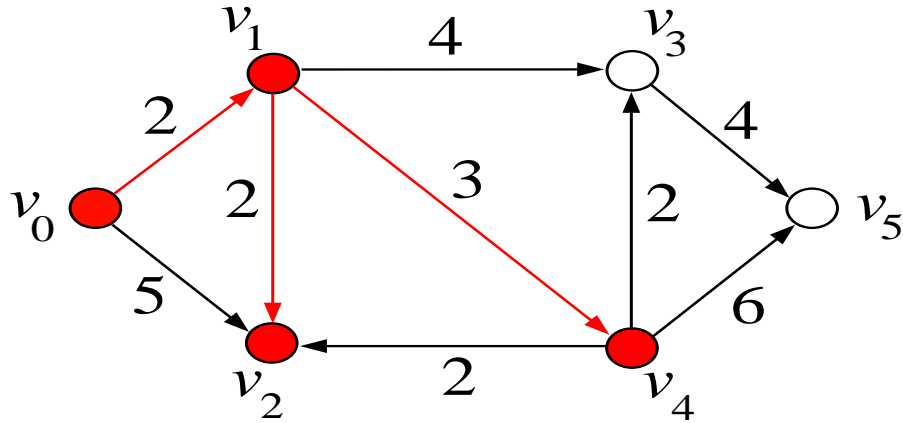
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1
2			$4/v_1$	$6/v_1$	$5/v_1$	∞	v_2
3				$6/v_1$	$5/v_1$	∞	

Dijkstra's Algorithm



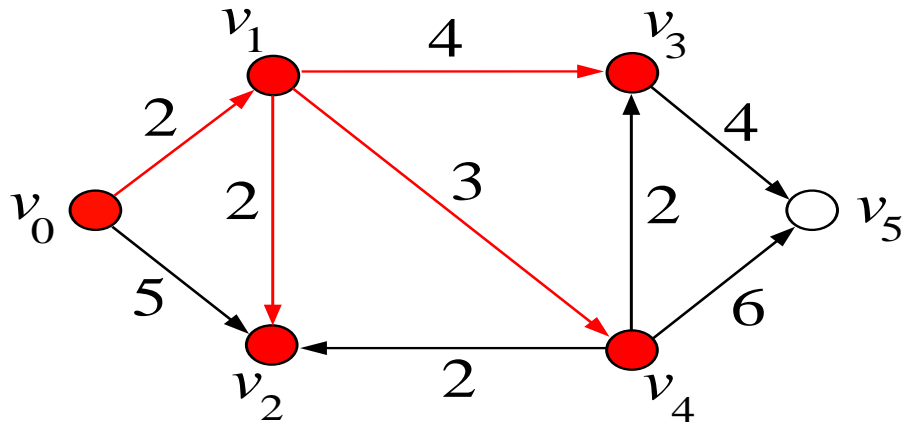
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1
2			$4/v_1$	$6/v_1$	$5/v_1$	∞	v_2
3				$6/v_1$	$5/v_1$	∞	v_4

Dijkstra's Algorithm



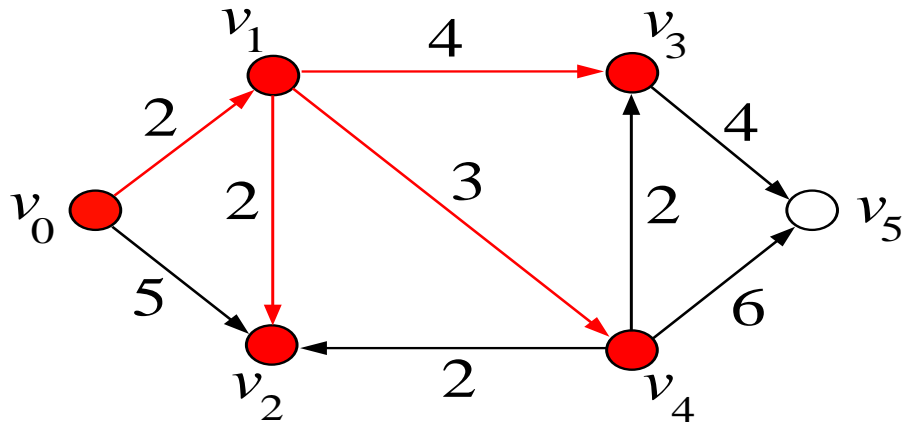
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1
2			$4/v_1$	$6/v_1$	$5/v_1$	∞	v_2
3				$6/v_1$	$5/v_1$	∞	v_4
4				$6/v_1$		$11/v_4$	

Dijkstra's Algorithm



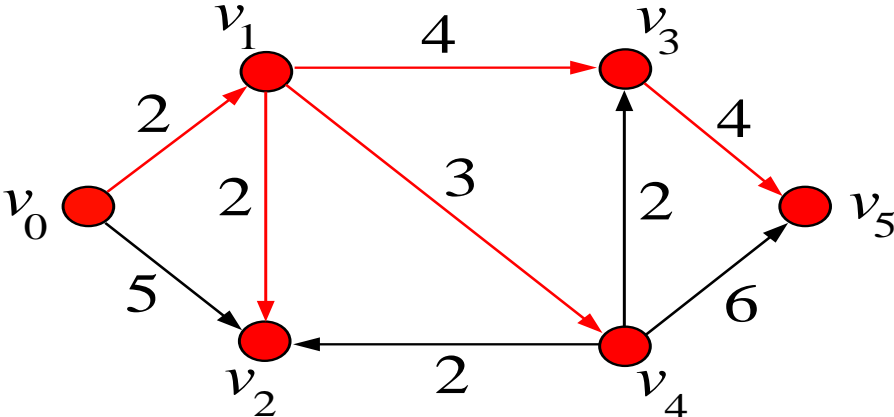
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1
2			$4/v_1$	$6/v_1$	$5/v_1$	∞	v_2
3				$6/v_1$	$5/v_1$	∞	v_4
4				$6/v_1$		$11/v_4$	v_3

Dijkstra's Algorithm



	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1
2			$4/v_1$	$6/v_1$	$5/v_1$	∞	v_2
3				$6/v_1$	$5/v_1$	∞	v_4
4				$6/v_1$		$11/v_4$	v_3
5						$10/v_3$	

Dijkstra's Algorithm



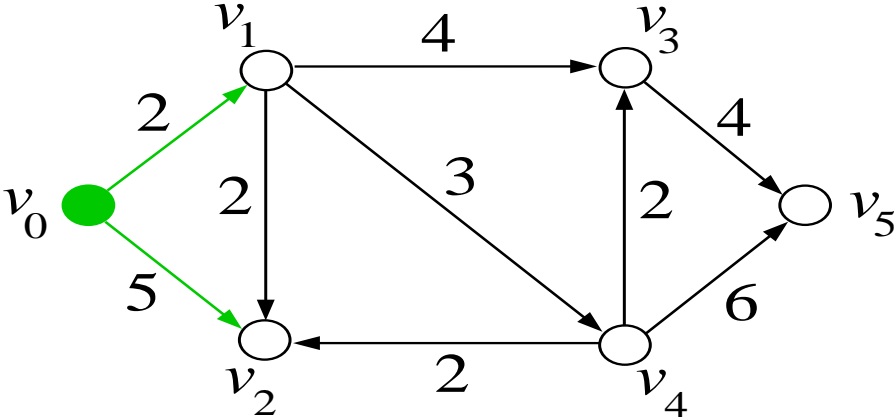
	v_0	v_1	v_2	v_3	v_4	v_5	chosen node
0	0	∞	∞	∞	∞	∞	v_0
1		$2/v_0$	$5/v_0$	∞	∞	∞	v_1
2			$4/v_1$	$6/v_1$	$5/v_1$	∞	v_2
3				$6/v_1$	$5/v_1$	∞	v_4
4				$6/v_1$		$11/v_4$	v_3
5						$10/v_3$	v_5

MOORE'S ALGORITHM

Moore's Algorithm

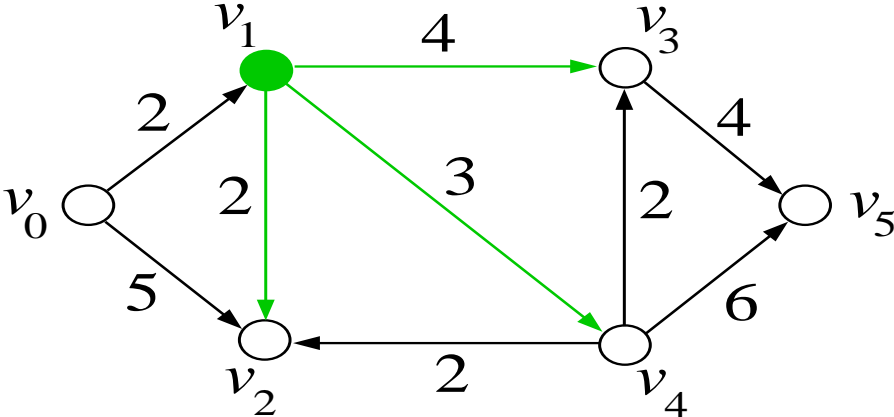
```
 $\ell(v_0) := 0; a(v_0) := 0; p(v_0) := *; S := 0; I := 1;$   
FOR  $v \in V \setminus \{v_0\}$  DO:  $\ell(v) := \infty; a(v) := \infty; p(v) := *;$  END;  
  
WHILE  $I = 1$  DO:  
   $I := 0;$   
  FOR  $v \in V$  DO:  
    IF  $a(v) = S$  THEN  
      FOR  $\bar{v} \in A_v$  DO  
        IF  $\ell(\bar{v}) > \ell(v) + w(v, \bar{v})$  THEN  
           $I := 1; \ell(\bar{v}) := \ell(v) + w(v, \bar{v}); a(\bar{v}) := a(v) + 1; p(\bar{v}) := v;$   
        END IF;  
      END;  
    END IF;  
  END;  
   $S := S + 1;$   
END;
```


Moore's Algorithm



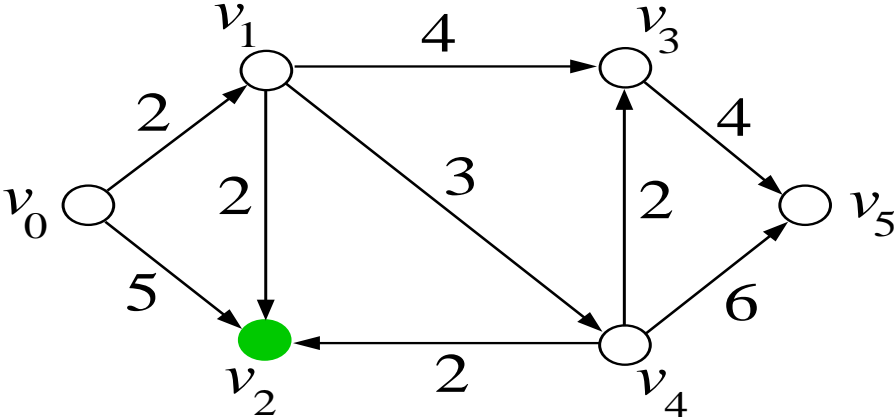
$v / l/a$	v_0	v_1	v_2	v_3	v_4	v_5	S	I
	0/0	∞/∞	∞/∞	∞/∞	∞/∞	∞/∞	0	0
v_0		2/1	5/1	∞/∞	∞/∞	∞/∞	0 \rightarrow 1	0 \rightarrow 1

Moore's Algorithm



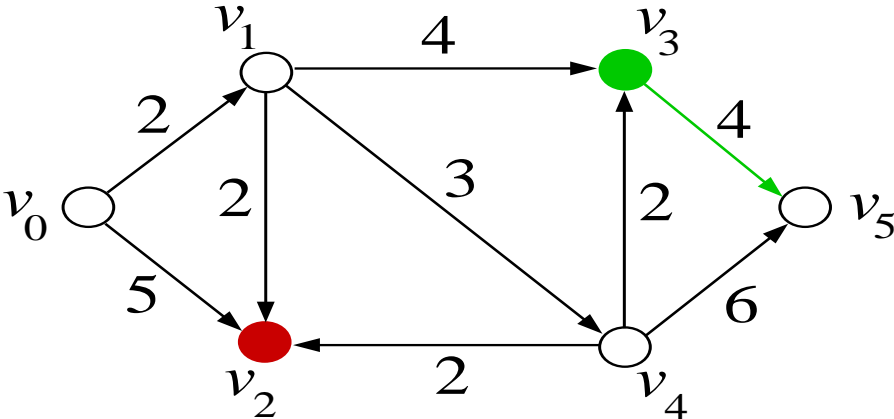
$v / l/a$	v_0	v_1	v_2	v_3	v_4	v_5	S	I
	0/0	∞/∞	∞/∞	∞/∞	∞/∞	∞/∞	0	0
v_0		2/1	5/1	∞/∞	∞/∞	∞/∞	0 \rightarrow 1	0 \rightarrow 1
v_1			4/2	6/2	5/2	∞/∞	1 \rightarrow 2	0 \rightarrow 1

Moore's Algorithm



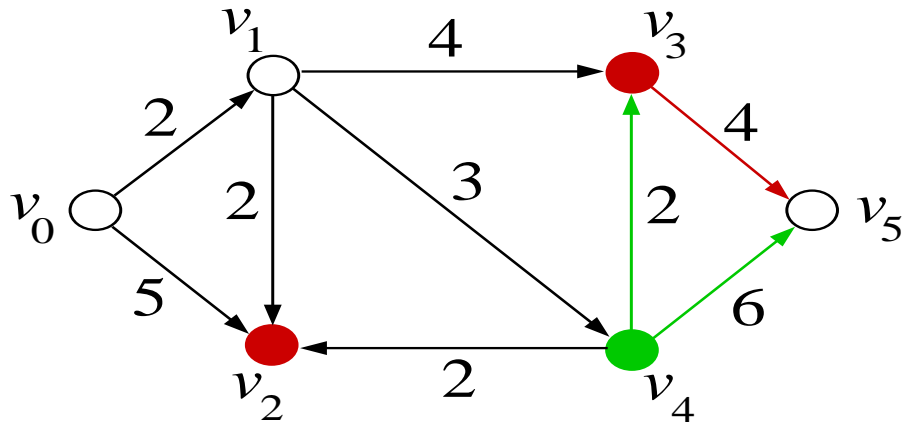
$v / l/a$	v_0	v_1	v_2	v_3	v_4	v_5	S	I
	0/0	∞/∞	∞/∞	∞/∞	∞/∞	∞/∞	0	0
v_0		2/1	5/1	∞/∞	∞/∞	∞/∞	$0 \rightarrow 1$	$0 \rightarrow 1$
v_1			4/2	6/2	5/2	∞/∞	$1 \rightarrow 2$	$0 \rightarrow 1$
v_2				6/2	5/2	∞/∞	2	0

Moore's Algorithm



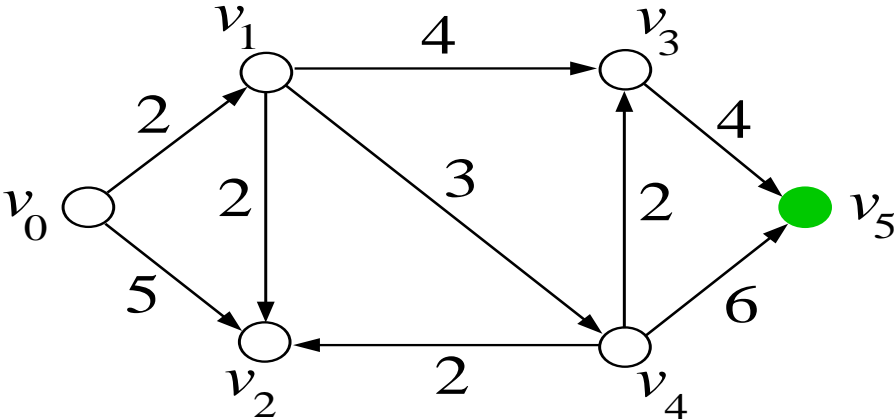
$v / l/a$	v_0	v_1	v_2	v_3	v_4	v_5	S	I
	0/0	∞/∞	∞/∞	∞/∞	∞/∞	∞/∞	0	0
v_0		2/1	5/1	∞/∞	∞/∞	∞/∞	$0 \rightarrow 1$	$0 \rightarrow 1$
v_1			4/2	6/2	5/2	∞/∞	$1 \rightarrow 2$	$0 \rightarrow 1$
v_2				6/2	5/2	∞/∞	2	0
v_3					5/2	10/3	2	$0 \rightarrow 1$

Moore's Algorithm



$v / l/a$	v_0	v_1	v_2	v_3	v_4	v_5	S	I
	0/0	∞/∞	∞/∞	∞/∞	∞/∞	∞/∞	0	0
v_0		2/1	5/1	∞/∞	∞/∞	∞/∞	$0 \rightarrow 1$	$0 \rightarrow 1$
v_1			4/2	6/2	5/2	∞/∞	$1 \rightarrow 2$	$0 \rightarrow 1$
v_2				6/2	5/2	∞/∞	2	0
v_3					5/2	10/3	2	$0 \rightarrow 1$
v_4						10/3	$2 \rightarrow 3$	1

Moore's Algorithm



$v / l/a$	v_0	v_1	v_2	v_3	v_4	v_5	S	I
	0/0	∞/∞	∞/∞	∞/∞	∞/∞	∞/∞	0	0
v_0		2/1	5/1	∞/∞	∞/∞	∞/∞	$0 \rightarrow 1$	$0 \rightarrow 1$
v_1		2/1	4/2	6/2	5/2	∞/∞	$1 \rightarrow 2$	$0 \rightarrow 1$
v_2		2/1	4/2	6/2	5/2	∞/∞	2	0
v_3		2/1	4/2	6/2	5/2	10/3	2	$0 \rightarrow 1$
v_4		2/1	4/2	6/2	5/2	10/3	$2 \rightarrow 3$	1
v_5		2/1	4/2	6/2	5/2	10/3	3	0

FLOYD-WARSHALL ALGORITHM

Floyd-Warshall Algorithm

$$V = \{v_1, v_2, \dots, v_n\}, \quad W = (w_{ij})_{1 \leq i, j \leq n}, \quad w_{ij} = w(v_i, v_j)$$

1. $l_{ij} := w_{ij}$

2. for $i=1$ to n do

 for $j=1$ to n do

 for $k=1$ to n do

$$l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$$

 end

 if $l_{jj} < 0$ then STOP (Cycle of negative length!)

 end

end

Floyd-Warshall Algorithm

$$L_0 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 8 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 2$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ \mathbf{2} & \mathbf{0} & \mathbf{8} & \infty & \mathbf{1} \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 2$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 8 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 2$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ \underline{2} & 0 & 8 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 2$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & \underline{0} & 8 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 2$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & \underline{8} & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 2$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \underline{\infty} & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```


Floyd-Warshall Algorithm

$$i = 1, j = 2$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & \underline{1} \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 2$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 3$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$i = 1, j = 3$ no improvement!

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 4$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ \underline{1} & \infty & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 4$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & \underline{\infty} & \infty & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 4$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & 3 & \underline{\infty} & 0 & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 4$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ \underline{1} & 3 & 5 & \underline{0} & 5 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```


Floyd-Warshall Algorithm

$$i = 1, j = 4$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & 3 & 5 & 0 & \underline{5} \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$i = 1, j = 4$$

$$L_0 \longrightarrow L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & 3 & 5 & 0 & 4 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

$$L_1 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ 2 & 0 & 6 & \infty & 1 \\ 6 & 2 & 0 & 4 & 3 \\ 1 & 3 & 5 & 0 & 4 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
end
```

Floyd-Warshall Algorithm

$$i = 2, j = 3$$

$$L_1 \longrightarrow L_2 = \begin{pmatrix} 0 & 2 & 4 & \infty & 3 \\ \mathbf{2} & \mathbf{0} & \mathbf{6} & \infty & \mathbf{1} \\ \mathbf{\underline{6}} & \mathbf{2} & \mathbf{0} & \mathbf{4} & \mathbf{3} \\ 1 & 3 & 5 & 0 & 4 \\ \infty & \infty & \infty & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
```

Floyd-Warshall Algorithm

Result:

$$L_5 = \begin{pmatrix} 0 & 2 & 4 & 4 & 3 \\ 2 & 0 & 6 & 2 & 1 \\ 4 & 2 & 0 & 4 & 3 \\ 1 & 3 & 5 & 0 & 4 \\ 2 & 4 & 6 & 1 & 0 \end{pmatrix}$$

```
for i=1 to n do
  for j=1 to n do
    for k=1 to n do
       $l_{jk} := \min(l_{jk}, l_{ji} + l_{ik})$ 
    end
    if  $l_{jj} < 0$  then STOP (Cycle of negative length!)
  end
end
end
```