# VU Diskrete Mathematik

# Exercises for Oct 13, 2023

**1)** A simple undirected graph is called cubic if each of its vertices has degree 3.

(a) Find a cubic graph with 6 vertices!

(b) Is there a cubic graph with an odd number of vertices?

(c) Prove that for all $n \geq 2$ there exists a cubic graph with $2n$ vertices!

**2)** Use a suitable graph theoretical model to solve the following problems:

(a) Show that in every city at least two of its inhabitants have the same number of neighbours!

(b) 11 friends want to send postcards according to the following rules: (i) Each person sends and receives exactly 3 cards. (ii) Each one receives only cards from those to whom he or she sent a card and *vice versa*.

Tell how this can be done or prove that this is impossible!

(c) Determine all graphs in which all vertices have degree 1.

(d) Determine all connected graphs having only vertices of degree 2.

**3)** Show that each of the following statements is equivalent to the statement "$T$ is a tree":

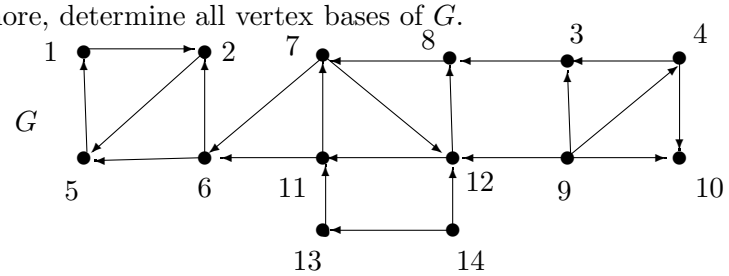
1. Every two nodes of $T$ are connected by exactly one path.
2. $T$ is connected and $\alpha_0(T) = \alpha_1(T) + 1$.
3. $T$ is a minimal connected graph, *i.e.*, deleting an edge destroys connectivity.
4. $T$ is a maximal acyclic graph, *i.e.*, adding an edge generates a cycle.

**4)** Show by induction that a connected graph on $n$ vertices has at least $n - 1$ edges.

**5)** Given the undirected graph $G = (V, E)$ with $V = \{a, b, c, d\}$ and $E = \{ab, ac, bd, cd\}$. Use the adjacency matrix to determine the number of walks of length four from $a$ to $d$.

**6)** Let $G = (V, E)$ be a simple, directed, and acyclic graph. Prove that $B = \{v \in V \mid d^-(v) = 0\}$ is a vertex basis of $G$. Furthermore, prove that $B$ is the only vertex basis of $G$.

**7)** Find the strongly connected components and the reduction $G_R$ of the graph $G$ below. Furthermore, determine all vertex bases of $G$.




**8)** Let $G = (V, E)$ be a simple and directed graph and $G_R$ its reduction. Prove that $G_R$ is acyclic!