Workshop on Dynamical Systems and Number Theory 2007

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Abstracts

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Entropy of conservative endomorphisms

Jon Aaronson
Tel Aviv University

We discuss entropy (as defined by Krengel in 1967) of infinite, conservative transformations introducing a similarity-invariant class of quasi-finite transformations (i.e. those for which the entropy of some set's 1st return time partition is finite). For these transformations there is a Pinsker- (i.e. maximal zero entropy-) factor generated by predictable sets and information convergence. In certain nice cases, we obtain distributional convergence of information. It turns out that there are probability preserving transformations with zero entropy with analogous properties. Joint work with Kyewon Koh Park.

On the discrepancy of $\{n_k x\}$

Christoph Aistleitner
Graz University of Technology

By a classical result of H. Weyl, for any increasing sequence (n_k) of integers, $\{n_k x\}$ is uniformly distributed mod 1 for almost every x in the sense of Lebesgue measure. For exponentially growing (n_k), W. Philipp (1975) proved the law of the iterated logarithm for the discrepancy of $\{n_k x\}$, but besides this, the order of magnitude of this discrepancy is known only for a few special sequences (n_k). In our lecture we prove an LIL type upper bound for subexponential growing (n_k) and extend our results to the discrepancy of $\{n_k x\}$ with multidimensional indices k. Also we sharpen Philipp's and Fukuyama's results in the exponential case by computing the precise constant in the LIL for a large class of sequences (n_k) and show that this constant is intimately connected with the number of solutions of the Diophantine equation $an_k + bn_l = c$.

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Some recent results and open problems in Ergodic Ramsey Theory

Vitaly Bergelson
Ohio State University

This partly expository talk will be devoted to a survey of some recent developments in Ergodic Ramsey Theory and to the discussion of natural open problems. The topics include:

- Ergodic Szemeredi theorem for general groups;
- Generalized polynomials and dynamical systems on nil-manifolds;
- Multiple recurrence and Hardy fields;
- Sumsets and Bohr sets.

On digits and mantissae in simple dynamical systems

Arno Berger
University of Canterbury (New Zealand), and
University of Alberta (Canada)

Significant digits and mantissae in sufficiently large and diverse aggregations of numerical data are often not uniformly but rather logarithmically distributed. This empirical observation, a notorious gem of popular mathematics usually referred to as Benford's Law (BL), has recently attracted interest from various mathematical disciplines, notably from stochastics, number theory, and dynamics. This talk will discuss some aspects of recent work on BL pertinent to (nonautonomous) dynamical systems, e.g. exponential dichotomies and scale distortion. A couple of challenging open questions will also be mentioned.

Brun's continued fraction algorithm and discrete geometry

Valérie Berthé
CNRS

The aim of this lecture is to show how to tackle two classical problems in discrete geometry by using a strategy based on multidimensional continued fractions. We will first discuss a discrete plane recognition algorithm connected with Brun's continued fraction algorithm. The problem of the discrete plane recognition consists in deciding whether a given set of points with integer coordinates can be described as a plane discretization. We will then consider the connectedness properties of arithmetic discrete hyperplanes.

A note on an ergodic theorem of Bourgain

András Biró
Alfréd Rényi Institute of Mathematics (Budapest)

Bourgain proved ergodic theorems for interesting arithmetical subsequences of the positive integers, e.g. for the primes or the squares. We show somewhat weaker statements for other subsequences (obtained by a sieving process).

On a mixed Littlewood conjecture in Diophantine approximation

Yann Bugeaud
Université Louis Pasteur (Strasbourg)

Recently, de Mathan and Teulié asked whether

$$\inf_{q\geq 1} q \cdot \|q\alpha\| \cdot |q|_p = 0 \tag{1}$$

holds for every badly approximable real number α and every prime number p. We survey recent results on this question, which is still unsolved. We give a simple combinatorial condition on the sequence of partial quotients of α under which (1) is true for every prime p. This is a joint work with Michael Drmota and Bernard de Mathan.

Topological factors of self-similar tiling systems

Fabien Durand
Université de Picardie Jules Verne (Amiens)

Given a non periodic self-similar tiling $\mathcal{T}$ generated by some similarity S_1 with stretching factor λ_1, it is rather natural to ask if we could generate $\mathcal{T}$ using another similarity with a different stretching factor λ_2. This is of course possible taking a power of the similarity S_1, where λ_2 is in this case a power of λ_1. C. Holton, C. Radin and L. Sadun showed in 2005 that the stretching factor of any other similarity which generates $\mathcal{T}$ is equal to a rational power of λ_1. More precisely, they prove that the stretching factors of conjugate tiling systems which are the orbit closure under Euclidean motions of some self similar tilings are multiplicatively dependent. In this paper we look at tiling systems which are the orbit closure under translations of some self similar tilings, in order to give a necessary condition to have non periodic common factors.

The problem we are interested in has been considered in 1969 by A. Cobham for fixed points of substitutions of constant length. He showed that if $p, q > 1$ are two multiplicatively independent integers then a sequence x on a finite alphabet is both p-substitutive and q-substitutive if and only if x is ultimately periodic, where p-substitutive means that x is the image by a letter to letter morphism of a fixed point of a substitution of constant length p. This theorem was the starting point of a lot of work in many different directions such as : numeration systems for $\mathbb{N}$, substitutive sequences and subshifts, automata theory and first order logic. Later, in 1977, A. Semenov proved a "multdimensional" Cobham type theorem, that is to say a Cobham theorem for recognizable subsets of $\mathbb{N}^d$. This result can be stated in terms of self similar tilings, and in the case these tilings are repetitive, our result is a generalization of Semenov Theorem.

This talk is based on joint work with M. I. Cortez.

Substitutions and the space filling property

Clemens Fuchs
ETH Zürich

It is well-known that to a given substitution σ a broken line can be associated that approximates an eigendirection of the coincidence matrix of σ and that by projecting this broken line to a hyperplane one obtains interesting structures. In this talk I will report on a joint paper with R. Tijdeman, where we do the projecting in such a way that we get a sequence of multi-dimensional words $w^{(n)}$. We associate to σ an abstract number system and relate its properties to properties of the words $w^{(n)}$. This is possible in very general cases, namely for unimodular primitive substitutions the characteristic polynomial of which has a dominant root $\beta > 1$ (we point out that the characteristic polynomial is not assumed to be irreducible). In particular, a criterion can be given which relates the structure of finite representations in this number system to the fact that the limit of $w^{(n)}$ fills up the full space and this criterion can be checked effectively if β is a Pisot number.

Enveloping semigroups in topological dynamics

Eli Glasner
Tel Aviv University

I'll survey the theory of enveloping semigroups in topological dynamics, beginning with the, already classical, theory of enveloping semigroups, due mainly to Robert Ellis. I'll then proceed to describe some new connections which were discovered in the last few years between three seemingly unrelated theories: of enveloping semigroups, of chaotic behavior, and of representation of dynamical systems on Banach spaces.

Computational aspects of symbolic dynamics

Michael Hochman
Hebrew University of Jerusalem

I will discuss recent developments about the descriptive complexity of multidimensional symbolic systems, such as shifts of finite type, sofic shifts and cellular automata. These results allow us characterize various properties of these systems, such as the numbers which can occur as entropies, and, to a large extent, the structure of the systems themselves.

Applications of harmonic analysis to measure rigidity for commuting automorphisms of the torus

Anatole Katok
Pennsylvania State University

Rigidity of invariant measures for algebraic actions of higher rank abelian groups has been an active research area during the last two decades. One of the principal cases concerns actions by commuting ergodic automorphisms of the torus. In this talk I will describe new results joint with Izabella Laba which establish measure rigidity in many situations not covered by the geometric method developed jointly with Ralf Spatzier in the mid-nineties. This gives an alternative approach to certain cases of results by M. Einsiedler and E. Lindenstrauss announced in 2003. Our approach is based on use of some harmonic analysis arguments inspired by an unpublished work of J. Feldman and M. Smorodinsky from 1992.

A conditionally sure ergodic theorem with an application to percolation

Michael Keane
Wesleyan University and Philips Research Laboratories

The traditional ergodic theorem provides, under stationarity, almost everywhere convergence of averages. In some situations it is desirable to have at one's disposal a type of convergence valid which holds (in a sense to be defined) everywhere; well-known is the case in which the function being averaged is a continuous function on a compact separable space equipped with a uniquely ergodic transformation. In this lecture we present a seemingly new theorem, which we call a conditionally sure ergodic theorem, and discuss our application of this theorem to solve an open problem in site percolation on "lattices with large holes". The method establishes uniqueness of the infinite cluster in the percolation regime for all such lattices in any dimension. No knowledge of percolation theory will be assumed. We also expect that the method will yield interesting number-theoretic applications. This is joint work with Masato Takei.

P-adic dynamical systems: cyclic and ergodic behavior

Andrei Khrennikov
University of Växjö

We plan recall (shortly) applications of p-adic numbers in theoretical physics: quantum theory and spin glasses, as well as in cognitive sciences and psychology. Then we shall present some mathematical results on ergodicity and cyclic behavior of p-adic and more general non-Archimedean dynamical systems, see e.g. [1]-[6] for details.

[1] A.Yu. Khrennikov, *p-adic valued distributions and their applications to the mathematical physics*, Kluwer, Dordreht, 1994.

[2] A.Yu. Khrennikov, *Non-Archimedean analysis: quantum paradoxes, dynamical systems and biological models.* Kluwer, Dordreht, 1997.

[3] A.Yu. Khrennikov, *Information dynamics in cognitive, psychological, social, and anomalous phenomena.* Kluwer, Dordreht, 2004.

[4] A.Yu. Khrennikov, M. Nilsson, *p-adic deterministic and random dynamical systems.* Kluwer-Springer, Dordreht, 2004.

[5] A. Yu Khrennikov, M. Nilsson, On the number of cycles for p-adic dynamical systems. *J. Number Theory*, **90**, 255-264 (2001).

[6] M. Gundlach, A. Khrennikov, K.-O. Lindahl, On ergodic behaviour of p-adic dynamical systems. *Inf. Dim. An., Quantum Prob. and Related Fields,* **4**, Nr. 4, 569-577 (2001).

The Skolem-Mahler-Lech theorem and dynamical systems

Jeffrey C. Lagarias
University of Michigan

The Skolem-Mahler-Lech theorem describes the zero set of a linear recurrence, and of power series coefficients of a rational function, over a field of characteristic zero. This talk describes extensions of the Skolem-Mahler-Lech theorem to polynomial dynamical systems acting on affine algebraic varieties. Such extensions were pioneered by Jason Bell, for polynomial automorphisms. The talk describes these results and further extensions to endomorphisms. (This is joint work in progress with Jason Bell.)

Chaos on hyperspaces

Marek Lampart
Silesian University in Opava

Let f be a continuous self–map of a compact metric space $\mathbb{X}$. The transformation f induces in a natural way a self–map $\overline{f}$ defined on the hyperspace $\mathcal{K}(\mathbb{X})$ of all nonempty closed subsets of $\mathbb{X}$. We study which of the most usual notions of chaos for dynamical systems induced by f are inherited by $\overline{f}$ and vice versa.

Firstly, we summarize the transmition of topological properties like transitivity and mixing. Secondly, we consider distributional chaos, Li–Yorke chaos, ω chaos, Devaney chaos, topological chaos (positive topological entropy), specification property and their variants. Finally, we answer questions stated independently by [J. Banks., Chaos Solitons Fractals, 25(3):681-685, 2005.] and by [H. Román-Flores, Chaos Solitons Fractals, 17(1):99-104, 2003.].

The homomorphism problem for $\beta\mathbb{N}$

Imre Leader
University of Cambridge

The space $\beta\mathbb{N}$ of ultrafilters on the natural numbers has a natural topological and algebraic structure. One of the most fascinating of the many unsolved problems about $\beta\mathbb{N}$ is the 'homomorphism problem': does there exist a non-trivial homomorphism from $\beta\mathbb{N}$ to itself? It turns out that this reduces to a very down-to-earth question about ultrafilters. After giving some background, we will discuss recent progress, including some links with a curious combinatorial problem.

Generating normal numbers over Gaussian integers

Manfred Madritsch
Graz University of Technology

This talk considers the construction of normal numbers in canonical number systems in the ring of Gaussian integers. As a way of construction the concatenation of integer parts of polynomials with complex coefficients is explained. It will be shown, that this construction constructs normal numbers in these number system. This is a generalization of methods used by Nakai and Shiokawa.

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Dirichlet series for finite combinatorial rank dynamics

Richard Miles
University of East Anglia

This talk will concern a class of group endomorphisms exhibiting slow orbit growth. An associated dynamical Dirichlet series is introduced and this is found to have a closed rational form. Analytic properties of the Dirichlet series are related to orbit-growth asymptotics: depending on the location of the abscissa of convergence and the degree of the pole there, various orbit-growth asymptotics are found, all of which are polynomially bounded.

On the non-monotonicity of the entropy of α-continued fraction transformations

Hitoshi Nakada
Keio University

We consider a one parameter family of maps of intervals called α-continued fraction transformations, $0 < \alpha < 1$. We denote by $H(\alpha)$ the entropy of the transformation (of index α) with respect to the absolutely continuous invariant measure. Recently Luzzi and Marmi showed that $H(\alpha)$ is continuous and goes to 0 as alpha tends to 0. We show the following:

(i) There exists positive constants C_1 and C_2 such that $C_1 < H(\alpha)/(-\log\alpha) < C_2$,

(ii) There exist sequences of intervals (I_n), (J_n), (K_n), and (L_n), $n > 2$, such that

- (a) $1/n$ is in I_n for $n > 2$,
- (b) $I_{n+1} < L_n < K_n < J_n < I_n$,
- (c) $H(\alpha)$ is increasing on I_n, decreasing on K_n , and constant on each J_n and L_n, $n > 2$,

(joint work with Rie Natsui).

The fine structure of dyadically badly approximable numbers

Johan Nilsson
Lund Institute of Technology

We consider badly approximable numbers in the case of dyadic diophantine approximation. For the unit circle $\mathbb{S}$ and the smallest distance to an integer $\|\cdot\|$ we give an elementary proof that the set $F_c = \{x \in \mathbb{S} : \|2^n x\| \geq c\,, n \geq 0\}$ is a fractal set whose Hausdorff dimension depends continuously on c, is constant on intervals which form a set of Lebesgue measure 1 and is self-similar. Hence it has a fractal graph. Moreover, the dimension of F_c is zero if and only if $c \geq 1 - 2\tau$, where τ is the Thue-Morse constant. We completely characterise the intervals where the dimension remains unchanged. As a consequence we can completely describe the graph of $c \mapsto \dim_H\{x \in [0,1] : \|x - \frac{m}{2^n}\| < \frac{c}{2^n} \text{ f.o.}\}$.

We also discuss the more general set $F_c^q = \{x \in \mathbb{S} : \|q^n x\| \geq c\,, n \geq 0\}$, for an integer $q \geq 2$, which corresponds to approximation with q-adic rationals. The set F_c^q shows a recursive behaviour in the sense that the graph of $c \mapsto \dim_H F_c^q$ contains the graph of $c \mapsto \dim_H F_c^{q-2}$.

Finally we say a few words of the simplified model, the one-sided approximation with dyadic, and the more general q-adic, rationals. We show that the one-sided case has similar properties to the two-sided; continuity, derivative zero Lebesgue a.e. and moreover we present how to characterise the intervals where the derivative is zero.

Thermodynamics of towers and the liftability problem

Yakov Pesin
Pennsylvania State University

Thermodynamical formalism of statistical physics is a collection of results aimed at producing some “natural” invariant measures with strong ergodic properties including Sinai-Ruelle-Bowen measures (absolutely continuous invarinant measures in the one-dimensional case), measures of maximal dimension and entropy. In particular, one can study the pressure function, which plays a crucial role in multifractal analysis and in applications to number theory. In the classical situations (Anosov maps, hyperbolic attractors, etc.) thermodynamical formalism can be effected using symbolic representations of these systems via subshifts of finite type. Many non-classical situations (e.g., one-dimensional unimodal maps and Henon attractors) can be handled using symbolic representations via towers whose base is the Bernoulli shift on a countable set of states. I will describe general tower constructions and present some recent results on thermodynamics of the corresponding systems and in particular, on the study of the pressure function. The principle new phenomenon is that one may have to reduce the class of invariant measures under consideration to the so-called liftable measures. I will describe these measures and discuss the associated liftability problem.

The Selberg zeta function via one dimensional dynamics

MARK POLLICOTT
Warwick University

It is well know that the Selberg zeta function for a compact Riemann surface has an analytic extension to the entire complex plane. Using thermodynamic ideas for interval maps it is possible to get more explicit information on the values it takes.

Meditations on the Bohr topology

IMRE Z. RUZSA
Alfréd Rényi Institute of Mathematics (Budapest)

In the *Bohr topology* on $\mathbb{Z}$, a basic neighbourhood of 0 is a set of the form

$$U(u_1, \dots, u_k, \varepsilon) = \{n \in \mathbb{Z} : \|nu_i\| < \varepsilon \text{ for } i = 1, \dots, k.\}$$

A classical theorem of Bogolyubov asserts that the set $A + A - A - A$ is a Bohr neighbourhood of 0 for every set A of positive (say, upper asymptotic) density.

This is a finite, effective result: there is an analog modulo m, and the density version follows (easily) from it. There is no topology modulo m, but we can define *Bohr* k, η*-subsets* of $\mathbb{Z}_m$ by

$$V(v_1, \dots, v_k, \eta) = \left\{n \in \mathbb{Z}_m : \left\|\frac{nv_i}{m}\right\| < \eta \text{ for } i = 1, \dots, k.\right\}.$$

The finite effective version asserts that for every set $A \subset \mathbb{Z}_m$, $|A| \geq \alpha m$ the set $S = A + A - A - A$ contains a Bohr k, η-set with an integer k and a positive η that depend on α only.

Instead of the four copies of A above, three suffice with a small change. Write $rA = \{rb : b \in A\}$. If r, s, t are nonzero integers satisfying $r + s + t = 0$ and A is a set of integers having positive upper density, then $S = rA + sA + tA$ is a Bohr neighbourhood of 0.

For $A + A - 2A$ this strengthens Bogolyubov's theorem; the proof is also finite and effective.

We cannot reduce three to two: $A - A$ may not be a Bohr neighbourhood of 0. This follows from a theorem of Kříž. There is, however, a weaker positive result, due to Følner: if A has positive density, then there is a Bohr neighbourhood U of 0 such that $U \setminus (A - A)$ has density 0.

We will retell Følner's proof in the following form. Let $A_m \subset \mathbb{Z}_m$, $|A_m| > \alpha m$. There is a k, an $\eta > 0$ and Bohr k, η-sets $V_m \subset \mathbb{Z}_m$ for *inifinitely many* values of m such that

$$|V_m \setminus (A_m - A_m)| = o(m)$$

(for these values of m).

Is this a finite effective theorem or not?

To formulate this exactly we define the *Bohr repulsion* of a set $A \subset \mathbb{Z}_m$ as

$$\text{br}(A, k, \eta) = \min\{|V \setminus A| : V \text{ is a Bohr } k, \eta \text{ set,}\}$$

and the *Bohr repulsion rate* as

$$\mathrm{brr}_m(\alpha, k, \eta) = \frac{1}{m} \max\{\mathrm{br}(A - A, k, \eta) : A \subset \mathbb{Z}_m, |A| \geq \alpha m\}.$$

With these notations the finite effective Følner theorem could sound as follows:

"For every $\alpha > 0$ there are k and $\eta > 0$ such that $\mathrm{brr}_m(\alpha, k, \eta) \to 0$ as $m \to \infty$."

We will give (not quite a complete proof, but at least) a convincing argument that the above statement fails for every $\alpha < 1/2$.

The following problems remain open:

In Følner's theorem, can one specify the k and ε in the description of U in terms of the density of A? Or at least k?

Følner's theorem easily implies that $A + A - A$ is a Bohr neighbourhood of almost all $a \in A$. Is this a finite effective theorem or not?

If $d(A) > 1/2$, then clearly $A - A = \mathbb{Z}$. In many questions on the difference set the answer is either "for all positive density", or "for no density $< 1/2$". Is there a reason for this? Is the following perhaps true?

"For every set A of positive density there are sets $B_1, \ldots, B_k$ such that $d(B_i) > 1/2 - \varepsilon$ and $\bigcap (B_i - B_i) \subset A - A$."

A general concept of Diophantine approximation

Jörg Schmeling
Lund University

We introduce a general scheme of generalized Diophantine approximations in metric spaces. We will discuss some general properties and then consider the problem on the circle. We will pose some questions and also apply some of the results to concrete situations. In particular we present a general principle that allows to analyze the dimension of sets that are infinitely or finitely often approximated with a given speed by a sequence (x_n) of points. This analysis will be in terms of the distribution of the sequence in the circle. This principle is motivated by dynamical properties of the sequence but also applicable to sequences that are not generated by a dynamical system. It is a generalization of the mass transference principle for mono-fractal measures by Beresnevich and Velani and the dynamical mass transference principle for multi-fractal measures by Fan, S., and Troubetzkoy.

2-dimensional continued fraction algorithms as dynamical systems

Fritz Schweiger
University of Salzburg

2-dimensional continued fractions are described by the following model. Let $B \subseteq \mathbf{R}^2$, I a countable index set, $B = \bigcup_{i \in I} B(i)$ a partition, and $T : B \to B$ a map such that the restriction of T to $B(i)$ is a fractional linear map. Two questions are immediate. Is T

ergodic with respect to Lebesgue measure λ? Does T admit a σ-finite invariant measure $\mu \sim \lambda$? Some examples lead to the following situation. Let R be a proper subset of I. Consider the set $\Gamma_R = \{x \in B : T^n x \in B(i), i \in R, n \geq 0\}$. The expected case is that Γ_R is of Cantor type with $\lambda(\Gamma_R) = 0$. However, the following situation may occur. For a suitable choice of R the set Γ_R consists of infinitely many segments of variable length which are glued together in one point but we have $\lambda(\Gamma_R) > 0$.

A geometric interpretation of the subword complexity

CHRISTIAN STEINEDER
Vienna University of Technology

Let $T : \mathbb{T}^d \to \mathbb{T}^d$, $T(x) = x + \vec{\alpha}$ be the ergodic group translation on the d–dimensional torus group $\mathbb{T}^d$ related to the classical sequence $n\vec{\alpha}$ (mod 1). Let $P \subset [0,1)^d$ be a convex polytope interpreted as a subset of $\mathbb{T}^d$ in the obvious way. T and P yield the binary coding sequence $\mathbf{h} \in \{0,1\}^{\mathbb{Z}}$, $h_k = 1$ iff $T^k(0) \in P$, $k \in \mathbb{Z}$. We want to present a geometric interpretation of the subword complexity of such a sequence $\mathbf{h}$ and consequently a connection between symbolic dynamics and convex geometry.

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Three dimensional symmetric Shift Radix Systems

PAUL SURER
University of Leoben

For $\mathbf{r} \in \mathbb{R}^{\mathbf{d}}$ define the mapping

$$\tau_{\mathbf{r},\varepsilon} : \mathbb{Z}^d \to \mathbb{Z}^d, \mathbf{x} = (x_1, \dots, x_d) \mapsto (x_2, \dots, x_d, -\lfloor \mathbf{r} \cdot \mathbf{x} + \varepsilon \rfloor).$$

$\tau_{\mathbf{r}}$ is called an ε-shift radix system (ε-SRS) if for all $\mathbf{x} \in \mathbb{Z}^d$ there exists an $n \in \mathbb{N}$ with $\tau^n_{\mathbf{r},\epsilon}(\mathbf{x}) = \mathbf{0}$. Originally shift radix systems have been introduced by Akiyama *et al.* with $\varepsilon = 0$. They are strongly related to other well known notions of number systems as β-expansion or canonical number. We will concentrate on the symmetric case ($\varepsilon = \frac{1}{2}$), which was the first time treated by Akiyama and Scheicher. Let

$$\begin{aligned} \mathcal{D}_d(\varepsilon) &:= \left\{ \mathbf{r} \in \mathbb{R}^d \,|\, \tau_{\mathbf{r},\varepsilon} \text{ is ultimately periodic} \right\} \text{ and} \\ \mathcal{D}^0_d(\varepsilon) &:= \left\{ \mathbf{r} \in \mathbb{R}^d \,|\, \tau_{\mathbf{r},\varepsilon} \text{ is an } \epsilon\text{-SRS} \right\}. \end{aligned}$$

The sets $\mathcal{D}_d(\varepsilon)$ are, except for the boundaries, easy to describe for all $\varepsilon \in [0, \frac{1}{2}]$ while the sets $\mathcal{D}^0_d(\epsilon)$ have a quite complicated structure, at least for $\varepsilon = 0$. Here, apart from the trivial cases $d = 0, 1$, exists only approximations for $d = 2$. In the symmetric case, the situation becomes more clearly. $\mathcal{D}^0_2(\frac{1}{2})$ has been fully analysed by Akiyama and Scheicher. We will supplement this result by giving a full characterisation of $\mathcal{D}^0_3(\frac{1}{2})$.

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A new look at the "Unsolved problems" of Paul R. Halmos

Benjamin Weiss
Hebrew University of Jerusalem

Last fall Paul Halmos, one of the pioneers of ergodic theory passed away at the age of 90. A little over 50 years ago he published a very influential introduction to ergodic theory which concluded with a list of ten unsolved problems. I will discuss the status of some of these with an emphasis on those for which solutions haven't appeared yet.