# EU Network in Noncommutative Geometry (EU-NCG)

## First Semester on Ergodic Theory and Operator Algebras

### Miniworkshop on Ergodic Theory and von Neumann Algebras December 3 - 14, 2007

As part of the first semester of the EU-NCG network based in Vienna there will be a workshop on connections between ergodic theory, dynamical systems and operator algebras. The workshop will be built around a small number of expository lecture courses, and will also include some seminars on current research.

The following main speakers are confirmed.

#### Christopher Deninger (University of Münster, Germany)

**Title**: Entropy of algebraic actions by amenable groups and determinants on Neumann algebras (4 lectures, December 10 - 14)

Abstract: We first discuss the notions of expansiveness and entropy for actions of amenable groups in measurable and topological dynamics. We then introduce a very interesting and explicit class of such dynamical systems called 'algebraic' which are constructed using ideals in group rings of amenable groups. The aim then is to decide expansiveness of these algebraic systems and to calculate the entropy of the actions. It turns out that the answers can be phrased in terms of the  $L^1$  group algebra and the von Neumann algebra of the amenable group. In particular the entropy is given by the logarithm of a suitable Fuglede-Kadison determinant. In the abelian case where the group is  $\mathbf{Z}^d$  this formula expresses entropy as a Mahler measure. We also explain how to calculate Fuglede-Kadison determinants explicitly in some non-commutative cases. At the end we will point out a *p*-adic analogue of the theory, relations to number theory and a relation to the theory of orthogonal polynomials on the unit circle. In the lectures we will make an effort to explain all the concepts and to point out some of the many open questions in this area.

#### Sergey Neshveyev (University of Oslo, Norway)

**Title**: Bost-Connes type systems, ergodic theory and equidistribution (5 lectures, December 3 – 7)

Abstract: In 1995 Bost and Connes constructed a remarkable  $C^*$ -dynamical system with the Galois group of the maximal abelian extension of  $\mathbf{Q}$  as the symmetry group and with phase transition related to properties of the Riemann zeta-function. Since then similar systems have been constructed for arbitrary number fields and for higher dimensions. The analysis of phase transition of these systems leads to study of actions of groups on adelic spaces. The uniqueness of a phase for large temperatures is related to ergodicity of certain actions and to equidistribution results in number theory. The goal of the lectures is to survey these developments.

### Ian Putnam (University of Victoria, Canada)

**Title**:  $C^*$ -algebras and topological dynamics (5 lectures, December 3 – 7)

**Abstract**: General theory: the construction of  $C^*$ -algebras from various types of dynamical systems: group actions, equivalence relations, topological groupoids. Basic properties of the  $C^*$ -algebras.

Specific applications: (topics chosen from) Orbit equivalence for minimal systems on Cantor sets, hyperbolic dynamical systems, aperiodic tilings.

### Erling Størmer (University of Oslo, Norway)

**Title**: Dynamical entropy in operator algebras (5 lectures, December 6 - 7 and 10 - 12)

Abstract: We first reformulate the definition of classical entropy and then show how the reformulation extends to a definition of noncommutative dynamical entropy. After having discussed its basic properties we consider the genralization of topological entropy to  $C^*$ -algebras. We shall then see how the two kinds of entropy behave under crossed products, the variational principle and binary shifts.

Further information will be posted as soon as it is available.

Young researchers (both pre- and postdocs) who are funded by the EU-NCG grant are expected to attend. They will be funded by the grant. There is some funding available from the ESI to help cover living expenses in Vienna for additional participants from outside the EU-Network.

Applications for participation and financial support should be sent to the ESI office (secr@esi.ac.at).

Klaus Schmidt