Minisymposium "Geometry, Analysis and Mathematical Physics"

CSASC 2011 Krems, 25-28 Sept 2011

List of talks:

María Barbero–Liñán

Institute for Mathematical Sciences (CSIC-UAM-UC3M-UCM), Madrid, Spain

email: mbarbero@icmat.es

Title: Lagrangian submanifolds as geometric tools to describe discrete Hamilton–Jacobi theory

Abstract: As Alan Weinstein stated once "everything is a Lagrangian submanifold". For instance, in 1976 Lagrangian submanifolds were used to describe Lagrangian mechanics by Tulczyjew. In classical mechanics Hamilton–Jacobi theory is useful to integrate partially or completely Hamilton's equations. Recent developments such as (Cariñena et al, 2006) have provided this theory with an intrinsic formulation. In this talk we will show how Lagrangian submanifolds offer a different interpretation of Hamilton–Jacobi theory in a continuous setting. Hamilton–Jacobi theory has also been studied in the discrete setting (Ohsawa, Bloch and Leok, 2009). Analogously to the above-mentioned descrip-

tion in the continuous setting, we present here a novel intrinsic description of discrete Hamilton–Jacobi theory in terms of Lagrangian submanifolds.

[joint work with Marina Delgado and David Martín]

****

Jaroslav Dittrich

Nuclear Physics Institute ASCR, CZ-250 68 Rez, Czech Republic

e-mail: dittrich@ujf.cas.cz

Title: Integer topological charges for finite energy fields in the O(3) sigma-model

Abstract: In the (2+1)-dimensional classical O(3) σ-model, all finite energy fields have integer topological charges regardless of their asymptotic behavior at infinity. Topological charge is conserved for the fields with finite Euclidean action, without further assumptions on the field equations or asymptotics. The fields with continuous first derivatives as well as fields in Sobolev-like spaces analogical to $W^{1,2}$ are considered.

****

Xavier Gràcia

Universitat Politècnica de Catalunya, Barcelona, Spain

email: xgracia@ma4.upc.edu

Title: The time-evolution operator for non-autonomous lagrangians in the homogeneous formalism

Abstract: The geometric description of time-dependent lagrangians can be done in the framework of jet bundles. In the singular case, it is also especially useful to consider the time -evolution operator that relates the constraints of hamiltonian and lagrangian formalisms. On the other hand, the homogeneous formalism transforms a time-dependent lagrangian into a time-independent homogeneous one. The main goal of this talk is to relate geometrically the dynamics and the constraints of these various settings.

****

Thomas Hoffmann-Ostenhof

University of Vienna, Austria; email: thoffmann@tbi.univie.ac.at

Title: Spectral Minimal Partitions

Abstract: Consider a domain $\Omega \subset \mathbb{R}^2$ and a partition of Ω into k pairwise disjoint sets, $D_1, D_2, \ldots, D_k$. Associate to each of the D_i the corresponding lowest Dirichlet eigenvalue $\lambda_1(D_i)$. Take the infimum of $\max_{i \leq k\, 1}(D_i)$ over all k-partitions and call it $\mathfrak{L}_k(\Omega)$. A partition for which the minimum a is achieved is called a spectral minimal partition.

In this talk we will review some properties of the $\mathfrak{L}_k$ and their associated $\mathcal{P}_k(\Omega)$.

In particular, relations of the $\mathfrak{L}_k(\Omega)$ with the spectrum of the Dirichlet Laplacian for Ω and the relation between the $\mathcal{P}_k(\Omega)$ with the eigenfunctions and their nodal domains are discussed. Finally, we present a new approach characterizing those $\mathfrak{L}_k(\Omega)$ and $\mathcal{P}_k(\Omega)$ with the help of Aharonov Bohm problems.

This is joint work with Bernard Helffer and Susanna Terracini; some important numerical work has been contributed by Virginie Bonnaillie Noel.

****

Author: Josef Janyška

Masaryk University, Brno, Czech Republic; e-mail: janyska@math.muni.cz

Title: Higher order Utiyama's reduction method

Abstract: Generalizations of Utiyama's reduction method will be presented for various types of connections (general linear connections on vector bundles, principal connections on principal bundles, general connections on fibered manifolds). In all cases the Utiyama's theorem and the invariant interaction can be generalized for any finite order.

****

Július Korbaš

Faculty of Mathematics, Physics, and Informatics, Comenius University, Bratislava,

Slovakia; e-mail: Julius.Korbas@fmph.uniba.sk

Title: On the characteristic rank of smooth manifolds

Abstract: The subject of this talk is the characteristic rank of smooth manifolds - a homotopy invariant introduced in J. Korbaš, The cup-length of the oriented Grassmannians vs a new bound for zero-cobordant manifolds, Bull. Belgian Math. Soc. 17 (2010), 69-81. More precisely, the characteristic rank, charrank(M), of a smooth closed connected d-dimensional manifold M is defined to be the largest integer k ($0 \leq k \leq d$) such that each element of the cohomology group $H^j(M; Z_2)$ with $j \leq k$ can be expressed as a polynomial in the Stiefel–Whitney characteristic classes of M.

****

Olga Krupkova

The University of Ostrava, Ostrava, Czech Republic La Trobe University, Melbourne, Australia

Title: Affine duality, and Lagrangian and Hamiltonian systems

Abstract: We use affine duals of jet-bundles to describe how Legendre maps may be used to provide Hamiltonian representations of variational problems in a single independent variable. Such a problem may given as a Lagrangian (of first-order or of higher-order), or alternatively as a locally variational form on a jet bundle

of arbitrary order with no preferred Lagrangian.

[joint work with David Saunders]

****

Eva Miranda

Universitat Politècnica de Catalunya, Barcelona, Spain

email: eva.miranda@upc.edu

Title: From action-angle coordinates to geometric quantization and back *Abstract:* A choice of polarization is a key ingredient in the geometric quantization scheme. A Lagrangian foliation gives a real polarization. It is a well-known result due to Snyaticki that in the case this regular Lagrangian is a fibration (with Hausdorff leaf space), the geometric quantization (à la Kostant) is given by its Bohr–Sommerfeld leaves. Under compactness assumptions, the set of Bohr–Sommerfeld leaves is discrete and can be determined via action-angle coordinates (Guillemin–Sternberg). In this talk, we will extend these results to the singular setting and explain some of our current projects. One of the main ingredients is the study of normal forms and action–angle coordinates with singularities for integrable systems in the symplectic and Poisson context.

****

Miguel-C. Muñoz–Lecanda

Universitat Politècnica de Catalunya, Barcelona, Spain

email: matmcml@ma4.upc.edu

Title: Kinematic reduction and the Hamilton–Jacobi theory

Abstract: The relationship between the classical Hamilton–Jacobi theory and the kinematic reduction of control systems by decoupling vector fields is studied in this talk. The geometric interpretation of this relationship relies on new mathematical techniques for mechanical systems defined on a skew-symmetric algebroid. This geometric structure allows us to describe nonholonomic systems, with both control and external forces, in a simplified way.

****

Jana Musilova

Masaryk University, Brno, Czech Republic; e-mail: janam@physics.muni.cz

Title: Particles with positive, negative, zero mass: a unified approach to relativistic mechanics within the non-holonomic variational theory

Abstract: Variational problems and variational theories with constraints are important in most physical disciplines. Very interesting questions appear in the context of non-holonomic mechanical systems, characterized by constraints on velocities. A typical nontrivial example of such a system is a particle in special relativity theory, where the well-known relation $u^i u^i = 1$ for four-velocity represents a nonlinear non-holonomic constraint. In a more general context, a

modification of this constraint can be used, allowing a unified approach to particles with all signs of mass - positive, negative and zero. Using this approach we can characterize all types of forces acting on a relativistic particle. It appears that we obtain not only the expected Lorentz-type force and, of course, constraint forces, but also the so called Dicke force proposed by Ernst Mach (Mach principle).

[joint work with Olga Krupkova]

****

David Saunders

The University of Ostrava, Ostrava, Czech Republic

e-mail: david@symplectic.demon.co.uk

Title: Homogeneity for higher-order ordinary differential equations *Abstract:* A system of homogeneous second order ordinary differential equations may be represented by a spray; the geodesics of the spray are curves in the base manifold with an orientation but without any particular parametrization. In this talk I shall describe a geometric homogeneity property for systems of higher-order ODEs, and show how the third order case differs from the case of fourth (and higher) order systems.

****

José A. Vallejo

Universidad Autónoma de San Luis Potosí, San Luis Potosí, Mexico

email: jvallejo@fc.uaslp.mx

Title: Lie algebroids of Poisson type

Abstract: I will show how to characterize those Lie algebroids on algebraic one-forms that come from Poisson algebras in a general setting (actually, that of Lie–Rinehart algebras). Moreover, I will give a description of the parametrization of a certain subclass of transitive algebroids by means of a connection and a way to construct new Poisson algebras from these transitive algebroids.

[http://arxiv.org/abs/1106.1512]