CSASC 2011: Minisymposium on Oscillation and Spectral Theory of Differential and Difference Equations

Organized by Ondřej Došlý, Roman Šimon Hilscher, and Gerald Teschl

Donau-Uni Krems
25.–28. September 2011

The purpose of the minisymposium is to bring together scientists working in the area of qualitative analysis of differential and difference equations. In particular, its aim is also to further strengthen the collaboration between mathematicians from the Czech Republic and Austria. Topics will include oscillation theory, spectral theory, indefinite operators, dynamic equations on time scales.

Schedule:

	Sunday 25.9	Monday 26.9	Wednesday 28.9
Coffee break			
10:30–11:00		Řehák	Grunert
11:00–11:30		Woracek	Behrndt
11:30–12:00		Eckhardt	Bognar
12:00–12:30		Teschl	Hilger
Lunch break			
15:00-15:30	Došlá		
15:30-16:00	Došlý		
16:00-16:30	Kratz		
16:30-17:00	Šimon Hilscher		
17:00-17:30	Ammann		

For further information on the conference see the official website:
`http://www.dmg.tuwien.ac.at/OMG/OMG-Tagung/`

Abstracts

Relative oscillation theory for Jacobi matrices

Kerstin Ammann
University of Vienna

Abstract. Classical oscillation theory for Jacobi matrices connects the number of eigenvalues below a given value with the number of sign flips of certain solutions of the underlying difference equation. Considered here will be the difference between the number of eigenvalues of two Jacobi matrices which we will connect with the number of sign flips of the Wronskian of two solutions of the underlying difference equations.

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Elliptic operators with δ-potentials

Jussi Behrndt
TU Graz

Abstract. We discuss some spectral properties of a class of uniformly elliptic second order differential operators with δ and δ'-potentials supported on smooth hypersurfaces.

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Spectral problems of some nonlinear partial differential equations

Gabriella Bognar
University of Miskolc

Abstract. Two eigenvalue problems will be considered:

$$\begin{array}{c} \operatorname{div}(|\nabla u|^{q-1}\nabla u) + \lambda |u|^{q-1} u = 0 \quad \text{in} \quad \Omega, \\ u = 0 \quad \text{on} \quad \partial\Omega, \end{array} \tag{1}$$

and

$$\begin{array}{c} \frac{\partial}{\partial x}\left(\left|\frac{\partial u}{\partial x}\right|^{q-1}\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{q-1}\frac{\partial u}{\partial y}\right) + \lambda |u|^{q-1} u = 0 \quad \text{in} \quad \Omega, \\ u = 0 \quad \text{on} \quad \partial\Omega, \end{array} \tag{2}$$

$0 < q < \infty$, and Ω is a bounded convex domain in $\mathbb{R}^2$.

The eigenvalues are examined for some special domains (for square and "circle" domains).

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Oscillation of four order differential equations

Zuzana Došlá
Masaryk University

Abstract. This is a joint work with M. Bartušek, M. Cecchi and M. Marini.

We study four order differential equations with the middle term

$$x^{(4)}(t) + q(t)x''(t) + r(t)f(x(t)) = 0 \tag{1}$$

as a perturbation of the linear equation

$$y^{(4)}(t) + q(t)y'' = 0. \tag{2}$$

We assume that the differential operator $L(u) = u'' + q(t)u$ is oscillatory, i.e. equation (1) cannot be written as a two-term equation.

Using a new iterative method, we show that for every solution y of (2) there exists a solution x of (1) such that $x^{(i)} - y^{(i)}$ $(i = 0, \dots, 3)$ have bounded variation in a neighborhood of infinity and tend to zero. In particular, we give conditions for the existence of bounded oscillatory solutions of (1). Our results are new also for linear equations.

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Two-parametric conditional oscillation of half-linear differential equations

Ondřej Došlý
Masaryk University, Brno

Abstract.

We introduce the concept of two-parametric conditional oscillation of the half-linear second order differential equation

$$\big(r(t)\Phi(x')\big)' + c(t)\Phi(x) = 0, \quad \Phi(x) := |x|^{p-2}x.$$

We show what role plays this concept in the half-linear oscillation theory.

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Measure Sturm–Liouville operators

Jonathan Eckhardt
University of Vienna

Abstract. Consider the measure Sturm–Liouville problem

$$-\frac{d}{dx}\frac{d}{d\varsigma}y(x) + \chi(x)y(x) = z\varrho(x)y(x), \quad x \in (a,b),\ z \in \mathbb{C}, \tag{1}$$

where ς, χ and ϱ are allowed to be locally finite signed Borel measures on some interval (a, b). We discuss self-adjointness and spectral theory for operators associated with this equation. This talk is based on joint work with G. Teschl.

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Transformation operators for Schrödinger operators on infinite-gap backgrounds

Katrin Grunert
Norwegian University of Science and Technology (NTNU)

Abstract. Transformation operators which preserve the asymptotic behavior at infinity are the main tool for considering different kinds of direct and inverse scattering problems. We present an investigation of the transformation operators for one-dimensional Schrödinger operators with potentials, which are asymptotically close to almost periodic infinite-gap potentials. At the end we will give an outlook on scattering theory in that case.

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Deformation of the Weyl algebra

Stefan Hilger
KU Eichstätt

Abstract. We will study the Weyl algebra and its so–called h– and q–deformations. These algebras provide an algebraic background for various types of differential and difference operators. The algebras will then appear in certain quiver representations that are connected to basic equations of mathematical physics.

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Spectral Theory of Continuous Hamiltonian and Discrete Symplectic Systems

Werner Kratz
University of Ulm, Germany

Abstract. We consider continuous Hamiltonian differential and discrete symplectic eigenvalue problems with Dirichlet boundary conditions. We present the basic results on these eigenvalue problems in both cases, continuous and discrete,

which are the *Oscillation Theorem*, *Rayleigh's Principle*, *Existence* of eigenvalues, the *Expansion Theorem* and *Completeness* of the eigenfunctions. The main tools for the proofs will be discussed, in particular: *Picone's Identity*, *l'Hospital's Rule* for matrices, and an *Index Theorem* for monotone matrix-valued functions.

[1] M. Bohner, O. Došlý and W. Kratz, Sturmian and spectral theory for discrete symplectic systems, TAMS 361(2009), 3109–3123.
[2] W. Kratz, Quadratic Functionals in Variational Analysis and Control Theory, Akademie Verlag 1995.

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A critical oscillation constant depends on time scales

Pavel Řehák
Academy of Sciences of the Czech Republic

Abstract. We discuss an extension of Hille-Nehari type criteria and Kneser type criteria to second order dynamic equations on arbitrary time scales. Both, linear and half-linear equations are considered. In particular, we get that the "magic" constant 1/4 known from the linear differential equations case (or the corresponding constant from the half-linear differential equations case) is not invariant with respect to the choice of a time scale. The results turn out to be new even in the well-studied difference equations case. Some applications, related results, and directions for a future research in this field will be indicated as well.

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Oscillation and spectral theory for Sturm–Liouville equations with nonlinearity in the spectral parameter

Roman Šimon Hilscher
Masaryk University

Abstract. We consider the second order Sturm–Liouville eigenvalue problem with Dirichlet boundary conditions and nonlinear dependence on the spectral parameter. We allow the potential to be (not necessarily strictly) monotone. We define new notions of eigenvalues and eigenfunctions and establish the corresponding oscillation theorem.

This work is based on a joint paper "Oscillation and spectral theory for linear Hamiltonian systems with nonlinear dependence on the spectral parameter"

(2011) with Martin Bohner (Missouri University of Science and Technology) and Werner Kratz (University of Ulm).

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On Sturm–Liouville operators on time scales

Gerald Teschl
University of Vienna

Abstract. We establish the connection between Sturm–Liouville equations on time scales and Sturm–Liouville equations with measure-valued coefficients. Based on this connection we are able to generalize several results for Sturm–Liouville equations on time scales which have been obtained by various authors in the past.

This talk is based on joint work with J. Eckhardt.

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An addendum to M.G.Kreĭn's Inverse Spectral Theorem for strings

Harald Woracek
TU Wien

Abstract. A string is a pair $(L, \mathfrak{m})$ where $L \in [0, \infty]$ and $\mathfrak{m}$ is a positive, possibly unbounded, Borel measure supported on $[0, L]$; we think of L as the length of the string and of $\mathfrak{m}$ as its mass density. To each string a boundary value problem is associated, namely

$$f'(x) + z \int_0^\infty f(y)\, d\mathfrak{m}(y),\ x \in \mathbb{R}, \qquad f'(0-) = 0\,.$$

A positive Borel measure τ on $\mathbb{R}$ is called a (canonical) spectral measure of the string $\mathrm{S}[L, \mathfrak{m}]$, if there exists an appropriately normalized Fourier transform of $L^2(\mathfrak{m})$ onto $L^2(\tau)$.

In order that a given positive Borel measure τ is a spectral measure of some string, it is necessary that:

- $\int_{\mathbb{R}} \frac{d\tau(\lambda)}{1+|\lambda|} < \infty$.
- Either $\operatorname{supp}\tau \subseteq [0, \infty)$, or τ is discrete and has exactly one point mass in $(-\infty, 0)$.

It is a deep result, going back to M.G.Kreĭn in the 1950's, that each measure with $\int_{\mathbb{R}} \frac{d\tau(\lambda)}{1+|\lambda|} < \infty$ and $\operatorname{supp}\tau \subseteq [0, \infty)$ is a spectral measure of some string, and that this string is uniquely determined by τ. The question remained open,

which conditions characterize whether a measure τ with $\operatorname{supp}\tau \not\subseteq [0,\infty)$ is a spectral measure of some string. In the present paper, we answer this question. Interestingly, the solution is much more involved than the first guess might suggest.

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