Boolean properties and Bell-like inequalities of numerical events

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Abstract

Let S be a set of states of a physical system and p(s) be the probability of the occurrence of an event when the system is in state $s \in S$. A function $p : S \to [0,1]$ is called a numerical event or alternatively an S-probability. If a set $P := \{p(s) \mid s \in S\}$ is ordered by the order of real functions such that certain plausible requirements are fulfilled, P becomes an orthomodular poset in which properties can be described by the addition and comparison of functions. P is then called an algebra of S-probabilities or algebra of numerical events. We first answer the question under which circumstances it is possible to consider sets of empirically found numerical events as members of an algebra of S-probabilities. Then we discuss the problem to decide whether a given small set P_n of S-probabilities can be embedded into a Boolean subalgebra of an algebra P of S-probabilities, in which case we will call P_n Boolean embeddable. If P_n is not Boolean embeddable, then the physical system at hand will most likely be non-classical. In the case of a concrete logic P, that is a quantum logic which can be represented by sets, we derive criteria for $P_n \subseteq P$ to be Boolean embeddable which can be checked by very simple procedures, for arbitrary S-probabilities we provide sets of Bell-like inequalities which characterize the Boolean embeddability of P_n. Finally we will show how these Bell-like inequalities fit into a general framework of Bell inequalities by providing a method for generating Bell inequalities for S-probabilities from elementary Bell valuations.