On ring-like structures of lattice-ordered numerical events

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Abstract

Let S be a set of states of a physical system. The probabilities $p(s)$ of the occurrence of an event when the system is in different states $s \in S$ define a function from S to $[0,1]$ called a numerical event or, more accurately, an *S-probability*. Sets of S-probabilities ordered by the partial order of functions give rise to so called algebras of S-probabilities, in particular to the ones that are lattice-ordered. Among these there are the σ-algebras known from probability theory and the Hilbert-space logics which are important in quantum-mechanics. Any algebra of S-probabilities can serve as a quantum-logic, and it is of special interest when this logic turns out to be a Boolean algebra because then the observed physical system will be classical. Boolean algebras are in one-to-one correspondence to Boolean rings, and the question arises to find an analogue correspondence for lattice-ordered algebras of S-probabilities generalizing the correspondence between Boolean algebras and Boolean rings. We answer this question by defining ring-like structures of events (RLSEs). First, the structure of RLSEs is revealed and Boolean rings among RLSEs are characterized. Then we establish how RLSEs correspond to lattice-ordered algebras of numerical events. Further, functions for associating lattice-ordered algebras of S-probabilities to RLSEs are studied. It is shown that there are only two ways to assign lattice-ordered algebras of S-probabilities to RLSEs if one restricts the corresponding mappings to term functions over the underlying orthomodular lattice. These term functions are the very functions by which also Boolean algebras can be assigned to Boolean rings.