Quantum measurements generating structures of numerical events

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Abstract

Let S be a set of states of a physical system and $p(s)$ the probability of an occurrence of an event when the system is in state $s \in S$. The function p from S to $[0,1]$ is called a numerical event, multidimensional probability or, more precisely, S-probability. If a set of numerical events is ordered by the order of real functions one obtains a partial ordered set P in which the sum and difference of S-probabilities are related to their order within P. According to the structure that arises this further opens up the opportunity to decide whether one deals with a quantum mechanical situation or a classical one. In this paper we focus on the situation that P is generated by a given set of measurements, i.e. S-probabilities, without assuming that these S-probabilities can be complemented by further measurements or are embeddable into Boolean algebras, assumptions that were made in most of the preceding papers. In particular, we study the generation by S-probabilities that can only assume the values 0 and 1, thus dealing with so called concrete logics. We characterize these logics under several suppositions that might occur with measurements and generalize our findings to arbitrary S-probabilities, this way providing a possibility to distinguish between potential classical and quantum situations and the fact that an obtained structure might not be sufficient for an appropriate decision. Moreover, we provide some explanatory examples from physics.