On bounded posets arising from quantum mechanical measurements

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Abstract

Let S be a set of states of a physical system. The probabilities $p(s)$ of the occurrence of an event when the system is in different states $s \in S$ define a function from S to $[0,1]$ called a numerical event or, more precisely, an *S-probability*. If one orders a set P of S-probabilities in respect to the order of functions, further includes the constant functions 0 and 1 and defines $p' = 1 - p$ for every $p \in P$, then one obtains a bounded poset of S-probabilities with an antitone involution. We study these posets in respect to various conditions about the existence of the sum of certain functions within the posets and derive properties from these conditions. In particular, questions of relations between different classes of S-probabilities arising this way are settled, algebraic representations are provided and the property that two S-probabilities commute is characterized which is essential for recognizing a classical physical system.

[1]Support of the research of the second author by the Austrian Science Fund (FWF), project I 1923-N25, and the Czech Science Foundation (GAČR), project 15-34697L, as well as by the project "Ordered structures for algebraic logic", supported by AKTION Austria – Czech Republic, project 75p11, is gratefully acknowledged.