Identifying quantum logics by numerical events

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Abstract

Let S be a set of states of a physical system and $p(s)$ the probability of an occurrence of an event when the system is in state $s \in S$. The function p from S to $[0,1]$ is called a numerical event, multidimensional probability or, more precisely, S-probability. Given a set of numerical events which has been obtained by measurements and not supposing any knowledge of the logical structure of the events that appear in the physical system, the question arises which kind of logic is inherent to the system under consideration. In particular, does one deal with a classical situation or a quantum one?

In this survey several answers are presented. Starting by associating sets of numerical events to quantum logics we study structures that arise when S-probabilities are partially ordered by the order of functions and characterize those structures which indicate that one deals with a classical system. In particular, sequences of numerical events are considered that give rise to Bell-like inequalities. At the center of all studies are so called algebras of S-probabilities, subsets of these and generalizations of them. A crucial feature of these structures is that order theoretic properties can be expressed by the addition and subtraction of real functions entailing simplified algorithmic procedures.

The study of numerical events and algebras of S-probabilities goes back to a cooperation of the physicist E. G. Beltrametti and the mathematician M. J. Mączyński in 1991 and has since then caused a series of subsequent papers of physical interest the main results of which will be summarized.