On a conjecture of L. Fejes Tóth and J. Molnár about circle coverings of the plane

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Abstract

In 1958 L. Fejes Tóth and J. Molnár formulated the conjecture that for a given homogeneity q the thinnest covering of the Euclidean plane by arbitrary circles is greater or equal a function S(q). In 1961 A. Florian proved that if the covering consists of only two kinds of circles then the conjecture is true supposed that $S(q) \leq S(1/q)$ what can be easily verified by a computer. In this paper we consider the general case of circles with arbitrary radii from an interval of the reals. We set up two further functions $M_0(q)$ and $M_1(q)$ and prove that the conjecture is true if S(q) is less than or equal to S(1/q), $M_0(q)$ and $M_1(q)$. As in the case of two kinds of circles this can be readily confirmed by computer calculations. (For $q \geq 0,6$ we even do not need the function $M_1(q)$ for computer aided comparisons.) Moreover, we obtain Florian's result in a shorter different way.