A paper begun as a student, completed as emeritus professor

Circle coverings, particularly the total covering of regions with circles of different sizes, have many practical applications, such as site selection for the construction of cell towers or location distributions in the transportation sector.
In 1958, the Hungarian mathematicians L. Fejes Tóth and J. Molnár conjectured that the thinnest covering of the plane with circles of arbitrary radii (i.e. one with the least overlapping of circles) is greater than or equal to a function S(q), where q is the infimum of the quotient of the radii of two circles within the covering (cf. [1]).
In 1961, A. Florian succeeded in proving this conjecture for the case when only circles with two distinct radii are allowed, provided that the function S(1/q) is greater than or equal to S(q), which can be easily verified numerically (cf. [2]).
As a student I attempted to prove the conjecture for arbitrary circle radii, which I did not succeed in doing, but which led to a result for a slightly different problem concerning circle coverings which I did not try to publish then. Only years later, when already working in graph theory and algebra I published this result (cf. [3]). As for the conjecture of L. Fejes Tóth and J. Molnár I had learned that various research groups had repeatedly achieved partial results but no proof and made up my mind to postpone dealing with this conjecture until I retire, and this I actually did.
As professor emeritus I first succeeded in finding upper and lower bounds for the function S(q) for which the work I had done as a student turned out to be decisive and after many further efforts I was finally successful in 2018 in proving the conjecture subject to the condition that three implicitly given functions in one variable are greater than or equal to another function in one variable, which can be easily verified numerically (see Figures below).

References

[1] L. Fejes Tóth and J. Molnár, Unterdeckung und Überdeckung der Ebene durch Kreise. Math. Nachr.18 (1958), 235-243
[2] A. Florian, Überdeckung der Ebene durch Kreise. Rend.Sem.Mat.Univ Padova 31 (1961), 77-86.
[3] D. Dorninger, Überdeckung der Ebene durch inkongruente Kreise. Elem. Math. 28 (1973),105-107
[4] D. Dorninger, Thinnest covering of the Euclidean plane with incongruent circles. Anal. Geom. Metric. Spaces 5 (2017), 40-46. DOI 10.1515/agms-2017-0002.
[5] D. Dorninger, On a conjecture of L. Fejes Tóth and J. Molnár about circle coverings of the plane. Period.Math.Hung. DOI: 10.1007/s10998-018-0254-z

Figures