

# Structural properties of algebras of $S$ -probabilities

Dietmar Dorninger and Helmut Länger

## Abstract

Let  $S$  be a set of states of a physical system. The probabilities  $p(s)$  of the occurrence of an event when the system is in different states  $s \in S$  define a function from  $S$  to  $[0, 1]$  called a *numerical event* or, more precisely, an  *$S$ -probability*. A set of  $S$ -probabilities comprising the constant functions 0 and 1 which is structured by means of the addition and order of real functions in such a way that an orthomodular partially ordered set arises is called an *algebra of  $S$ -probabilities*, a structure significant as a quantum-logic with a full set of states. The main goal of this paper is to describe algebraic properties of algebras of  $S$ -probabilities through operations with real functions. In particular, we describe lattice characteristics and characterize Boolean features. Moreover, representations by sets are considered and pertinent examples provided.

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