

Structural properties of algebras of S -probabilities

Dietmar Dorninger and Helmut Länger

Abstract

Let S be a set of states of a physical system. The probabilities $p(s)$ of the occurrence of an event when the system is in different states $s \in S$ define a function from S to $[0, 1]$ called a *numerical event* or, more precisely, an *S -probability*. A set of S -probabilities comprising the constant functions 0 and 1 which is structured by means of the addition and order of real functions in such a way that an orthomodular partially ordered set arises is called an *algebra of S -probabilities*, a structure significant as a quantum-logic with a full set of states. The main goal of this paper is to describe algebraic properties of algebras of S -probabilities through operations with real functions. In particular, we describe lattice characteristics and characterize Boolean features. Moreover, representations by sets are considered and pertinent examples provided.

¹Support of the research of the second author by the Austrian Science Fund (FWF), project I 1923-N25, and the Czech Science Foundation (GAČR), project 15-34697L, as well as by the project “Ordered structures for algebraic logic”, supported by AKTION Austria – Czech Republic, project 75p11, is gratefully acknowledged.