ON THE ERDŐS-SZEKERES CONVEX POLYGON PROBLEM

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The classic 1935 paper of Erdős and Szekeres entitled "A combinatorial problem in geometry" was a starting point of a very rich discipline within combinatorics: Ramsey theory. In that paper, Erdős and Szekeres studied the following geometric problem. For every integer $n \geq 3$, determine the smallest integer $ES(n)$ such that any set of $ES(n)$ points in the plane in general position contains n members in convex position, that is, n points that form the vertex set of a convex polygon. Their main result showed that $ES(n) \leq \binom{2n-4}{n-2} + 1 = 4^{n-o(n)}$. In 1960, they showed that $ES(n) \geq 2^{n-2} + 1$ and conjectured this to be optimal. In this talk, I will sketch a proof showing that $ES(n) = 2^{n+o(n)}$.