On the centre and commutator subgroup of finite groups

Gil Kaplan
School of Computer Sciences
The Academic College of Tel-Aviv-Yaffo

Abstract

For any group G, the centre $Z(G)$ and the commutator subgroup G' are subgroups which, in a sense, measure how close is the group to commutativity. Intuitively, we expect the size of G' to be "large", when the size of $Z(G)$ is "small". However, this connection does not hold in general. We obtain conditions on finite groups ensuring the inequality $|G'| > [G : Z(G)]^{1/2}$. In particular, this holds when the Frattini subgroup of G is trivial. In this case the above bound on the size of G' in terms of the size of $Z(G)$ is best possible.