Polynomial operators on varieties of languages and monoids

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Extended Abstract

One can assign to each positive variety $\mathcal{V}$ of regular languages and a fixed natural number k the class of all (positive) boolean combinations of the restricted polynomials, i.e. the languages of the form $L_0 a_1 L_1 a_2 \ldots a_\ell L_\ell,$ where $\ell \leq k$, $a_1, \ldots, a_\ell$ are letters and $L_0, \ldots, L_\ell$ are languages from the variety $\mathcal{V}$. The resulting classes $\mathsf{PPol}_k\mathcal{V}$ and $\mathsf{BPol}_k\mathcal{V}$ are positive and boolean varieties, respectively.

In the case that the pseudovariety $\mathbf{V}$ of finite ordered monoids corresponding to $\mathcal{V}$ consists of finite members of a variety $\mathsf{Var}\,\mathcal{V}$ of ordered monoids (that is, $\mathsf{Var}\,\mathcal{V} = \mathsf{HSP}\,\mathbf{V}$ and $\mathbf{V} = \mathsf{Fin}(\mathsf{Var}\,\mathcal{V})$) which is locally finite, we can effectively describe the fully invariant compatible quasiorder on X^* (where $X = \{x_1, x_2, \ldots\}$ are variables) for $\mathsf{Var}(\mathsf{PPol}_k\mathcal{V})$ and $\mathsf{Var}(\mathsf{BPol}_k\mathcal{V})$ in terms of that for $\mathbf{V}$.

The paper [2] deals with the operator on the class $\mathcal{V}_0$ of languages where $\mathcal{V}_0(A) = \{\emptyset, A^*\}$ for each finite alphabet A. They form two natural hierarchies within piecewise testable languages and the boolean case has been studied in papers by Simon [5], Blanchet-Sadri [1], Volkov [6] and others. The main issues were the existence of finite bases of identities for the corresponding pseudovarieties of monoids and generating monoids for these pseudovarieties. In [2] we studied mainly the positive case and even finite unions of $A^* a_1 A^* a_2 A^* \ldots A^* a_\ell A^*,$ where $a_1, \ldots, a_\ell \in A,\ \ell \leq k.$

In [3] we rather considered the operators on classes of languages and the corresponding operator on classes of ordered monoids in a general case. We characterized in various terms varieties which are generated by a finite number of languages.

In [4] we study four hierarchies of languages which result by applying the restricted positive or boolean polynomial operator to the positive varieties where the class $\mathcal{V}(A)$ equals to finite unions of $B^*,\ B \subseteq A$ or $\mathcal{V}(A)$ equals to finite unions of $\overline{B},\ B \subseteq A$ where $\overline{B}$ is the set of all words over A containing exactly the letters from B. Our basic questions are to explore all the inclusions among our varieties and we start to discuss the existence of finite bases for corresponding pseudovarieties of (ordered) monoids. Hopefully our results bring a bit more light into the complexity of the structure of (positive) subvarieties of the second level of the Straubing-Thérien hierarchy.

In our talk we are going to concentrate on the monoid side, we will mainly discuss the identities for our varieties, relatively free monoids. and a possibility of a generating by a single (ordered) monoid.

References

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