

FORD-FULKERSON ALGORITHM

Ford-Fulkerson Algorithm

FOR $e \in E$ DO: $\phi(e) := 0$: END; $P := \text{true}$;

WHILE $P = \text{true}$ DO:

$p(s) := -$; $\delta(s) := \infty$; $V_1 := \{s\}$; $V_2 := V \setminus \{s\}$; $S := \emptyset$;

WHILE $|V_1| > |S|$ DO:

$S := V_1$;

FOR $x \in V_1$ DO:

FOR $y \in \Gamma^+(x) \cap V_2$ DO:

IF $\phi(\langle x, y \rangle) < w(\langle x, y \rangle)$ THEN

$p(y) := +x$; $\delta(y) := \min(\delta(x), w(\langle x, y \rangle) - \phi(\langle x, y \rangle))$;

$V_1 := V_1 \cup \{y\}$; $V_2 := V_2 \setminus \{y\}$;

END IF;

END;

FOR $y \in \Gamma^-(x) \cap V_2$ DO:

IF $\phi(\langle y, x \rangle) > 0$ THEN

$p(y) := -x$; $\delta(y) := \min(\delta(x), \phi(\langle x, y \rangle))$;

$V_1 := V_1 \cup \{y\}$; $V_2 := V_2 \setminus \{y\}$;

END IF;

END;

END;

END;

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IF  $t \in V_2$  THEN  $P := \text{false}$ ;

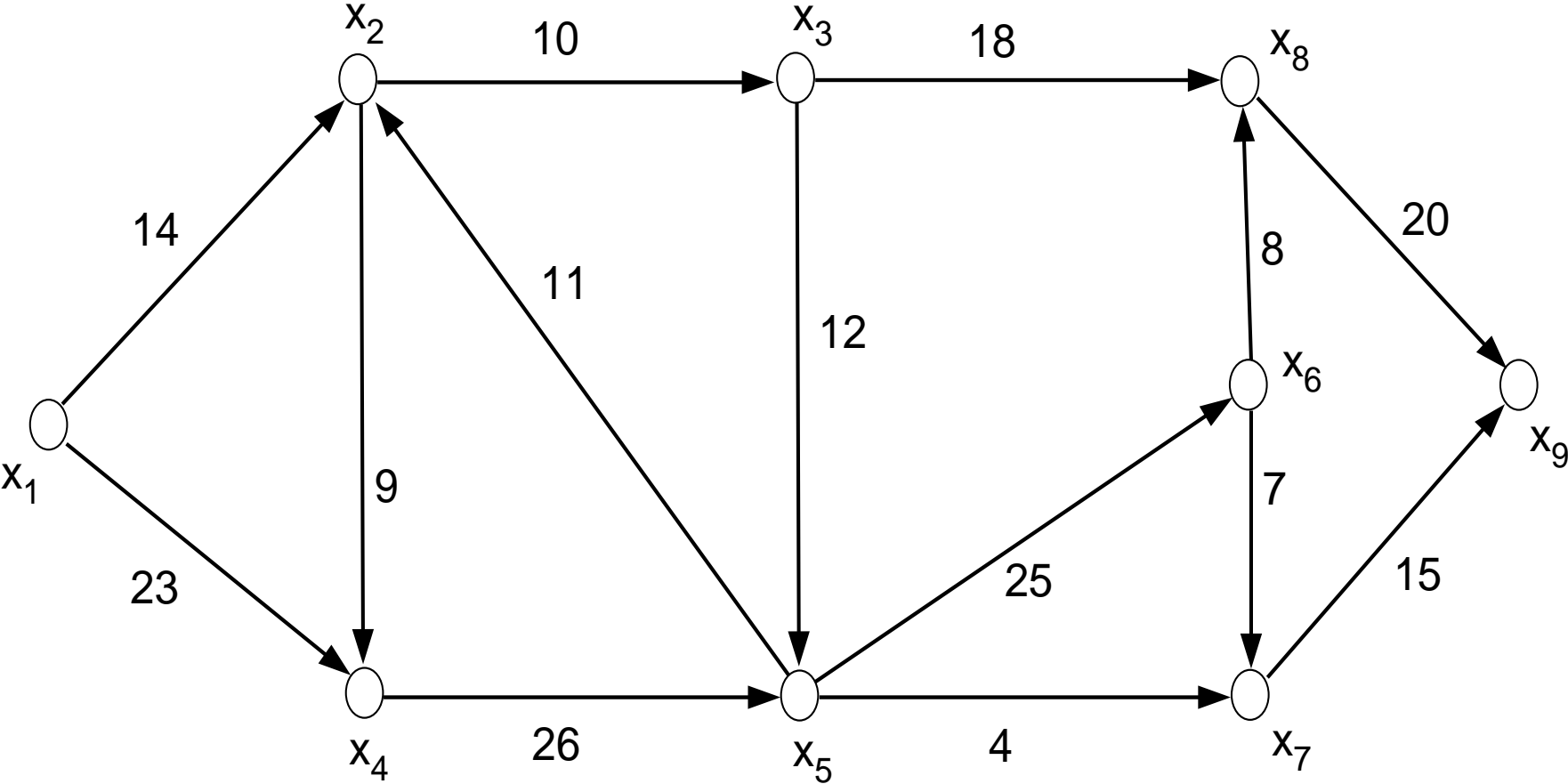
ELSE

     $x := t$ ;
    WHILE  $x \neq s$  DO:
        IF  $p(x) = +z$  THEN  $\phi(\langle z, x \rangle) := \phi(\langle z, x \rangle) + \delta(t)$ ;     $x := z$ ;
        END IF;
        IF  $p(x) = -z$  THEN  $\phi(\langle x, z \rangle) := \phi(\langle x, z \rangle) - \delta(t)$ ;     $x := z$ ;
        END IF;
    END;

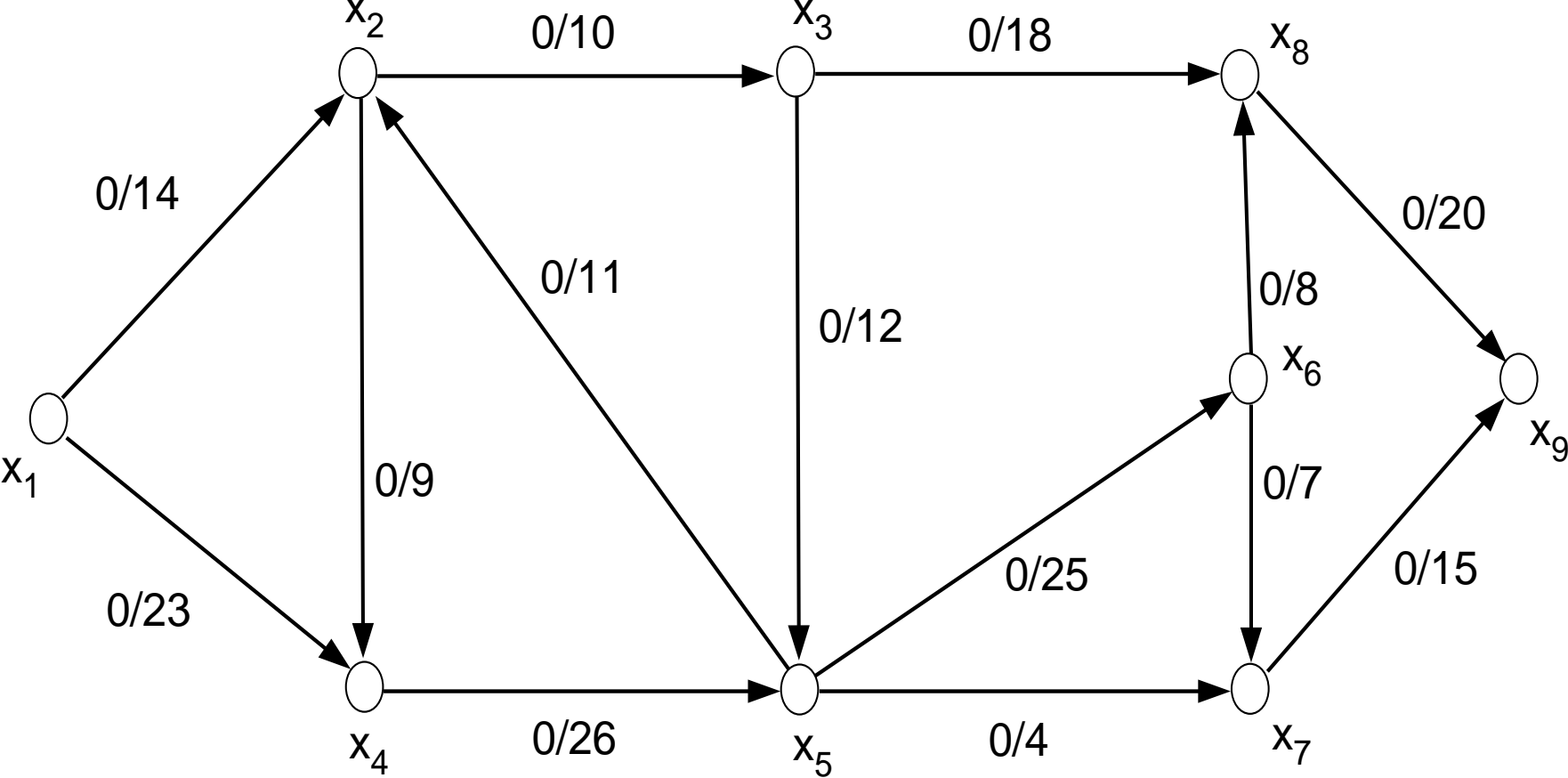
END IF;
END;

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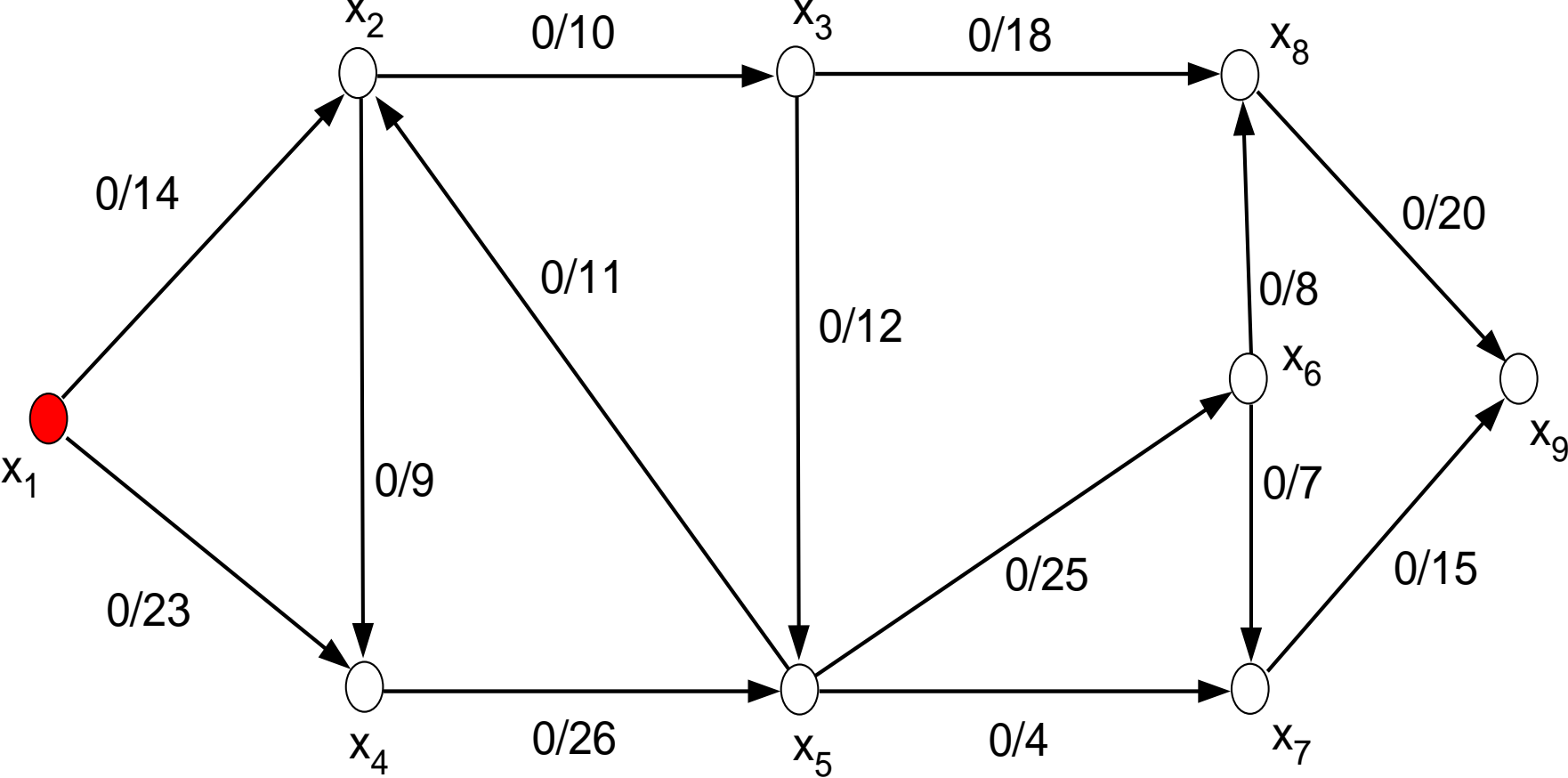
Ford-Fulkerson Algorithm



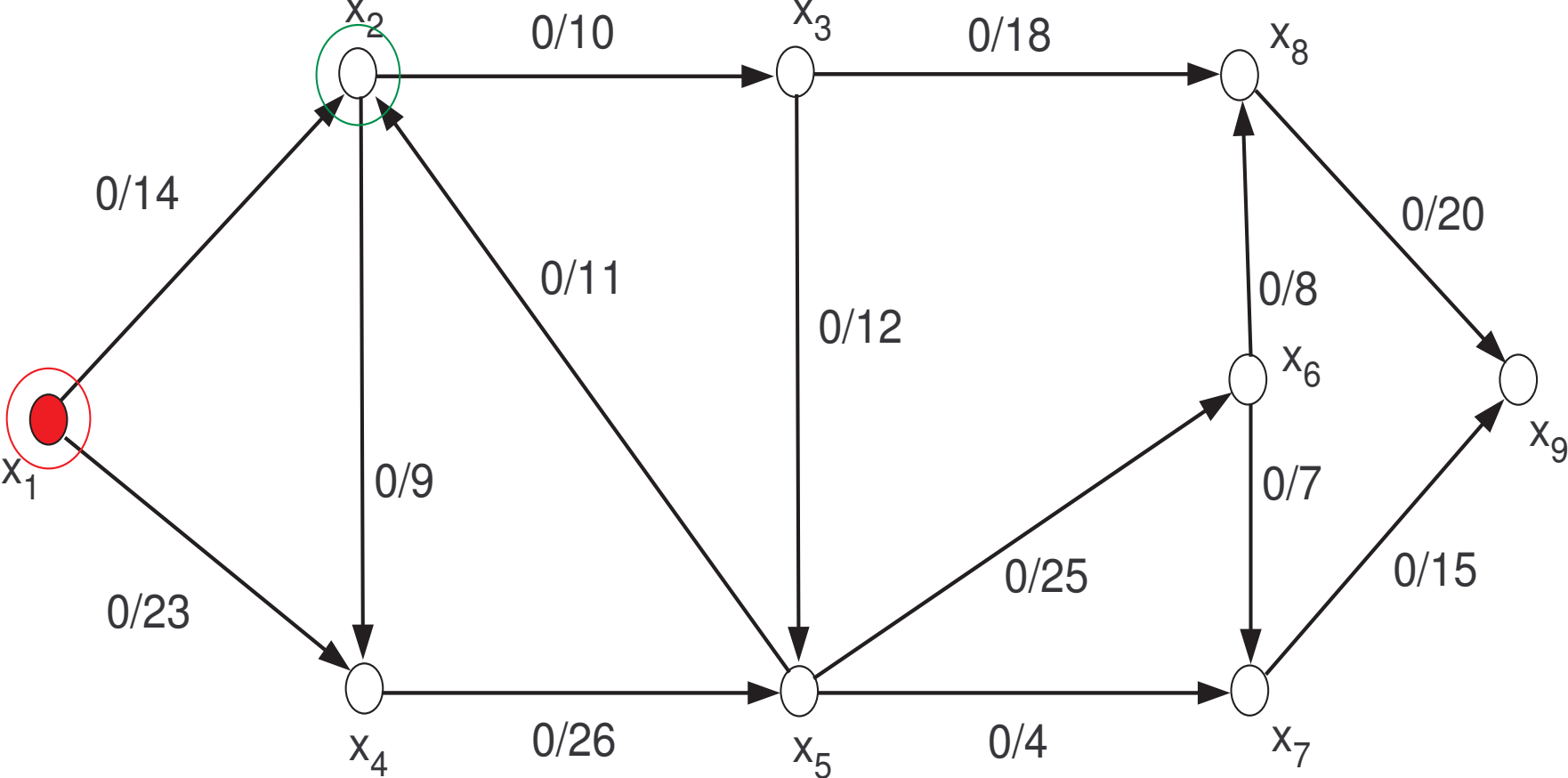
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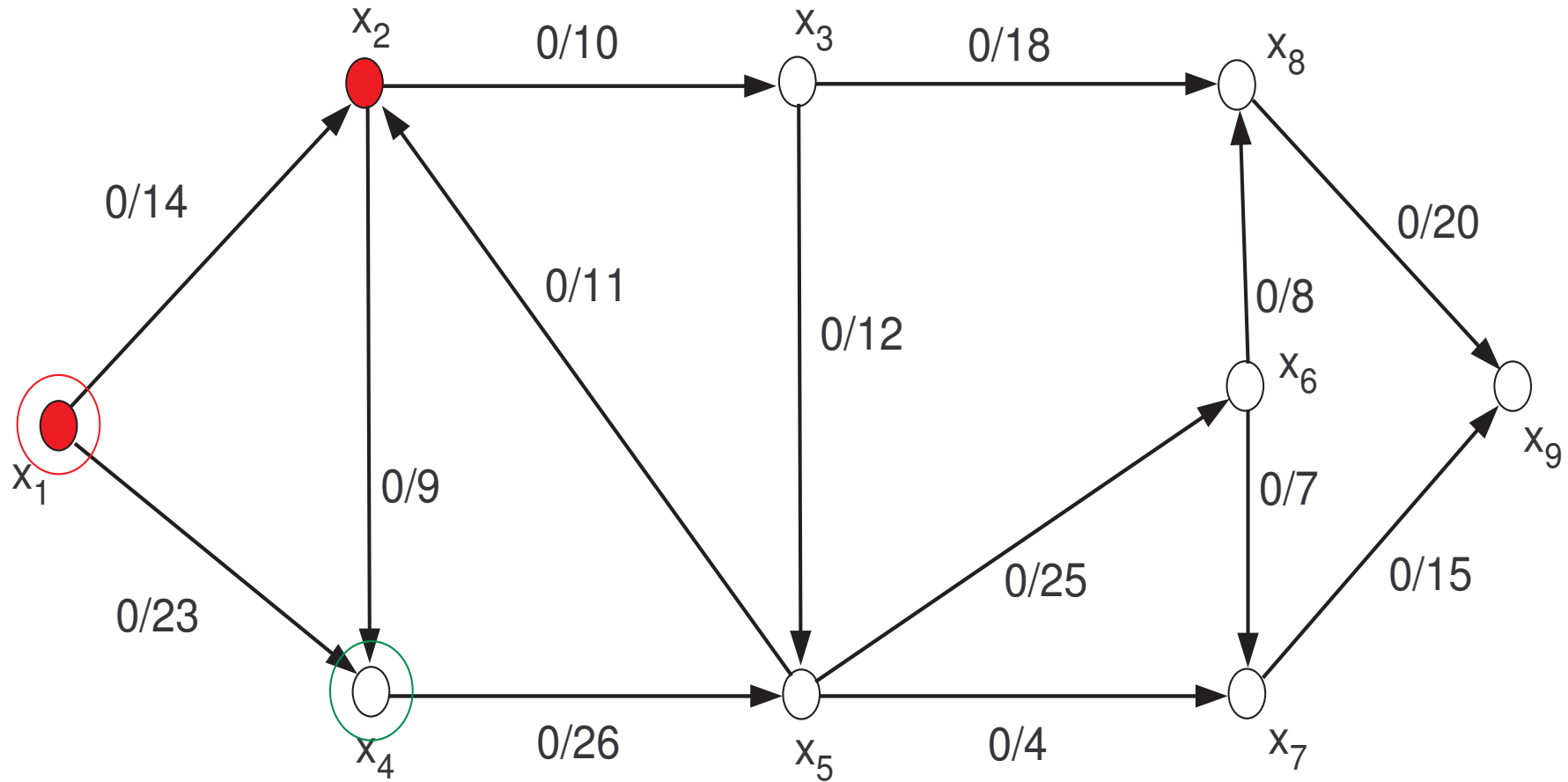
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Ford-Fulkerson Algorithm

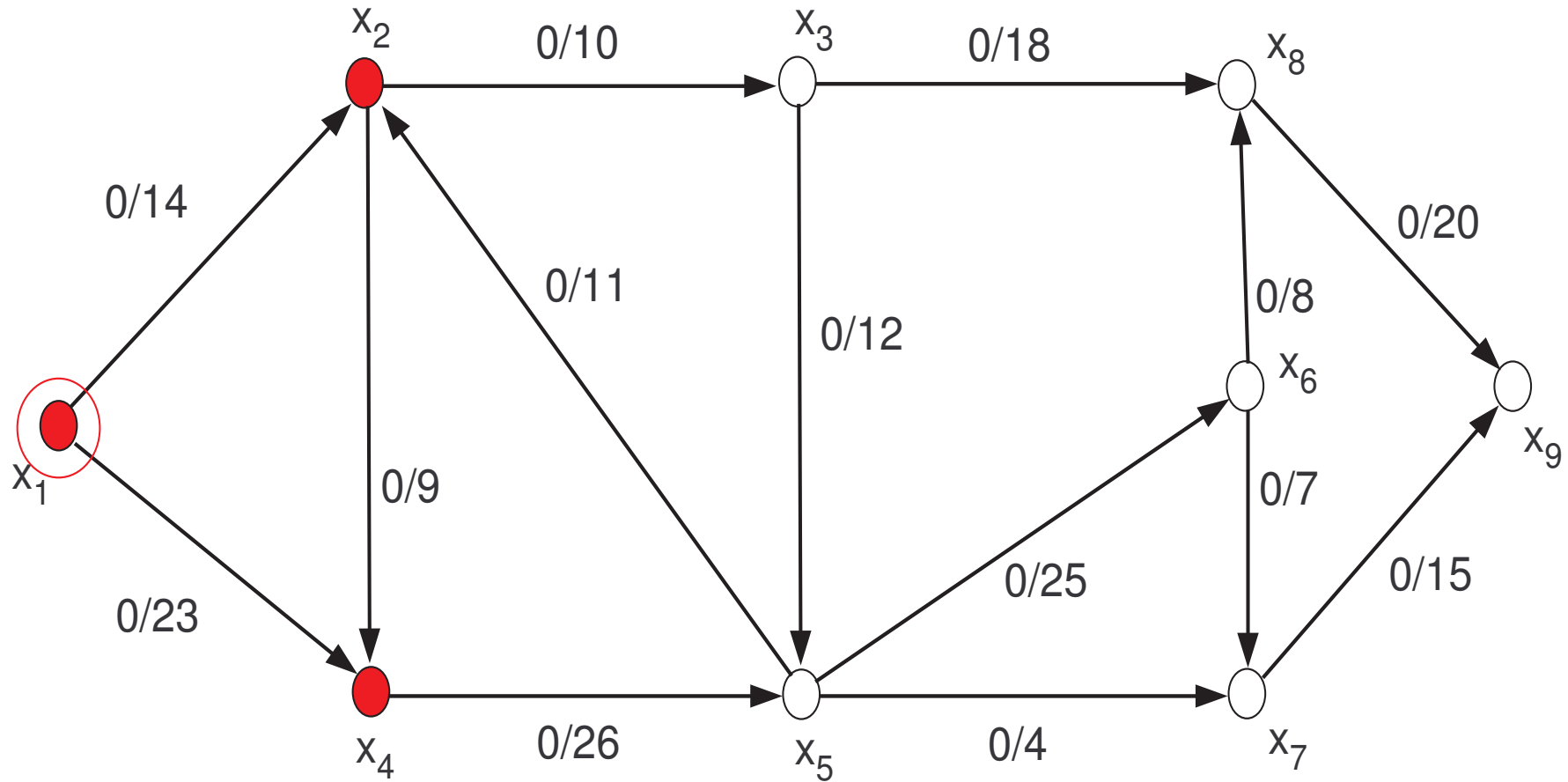


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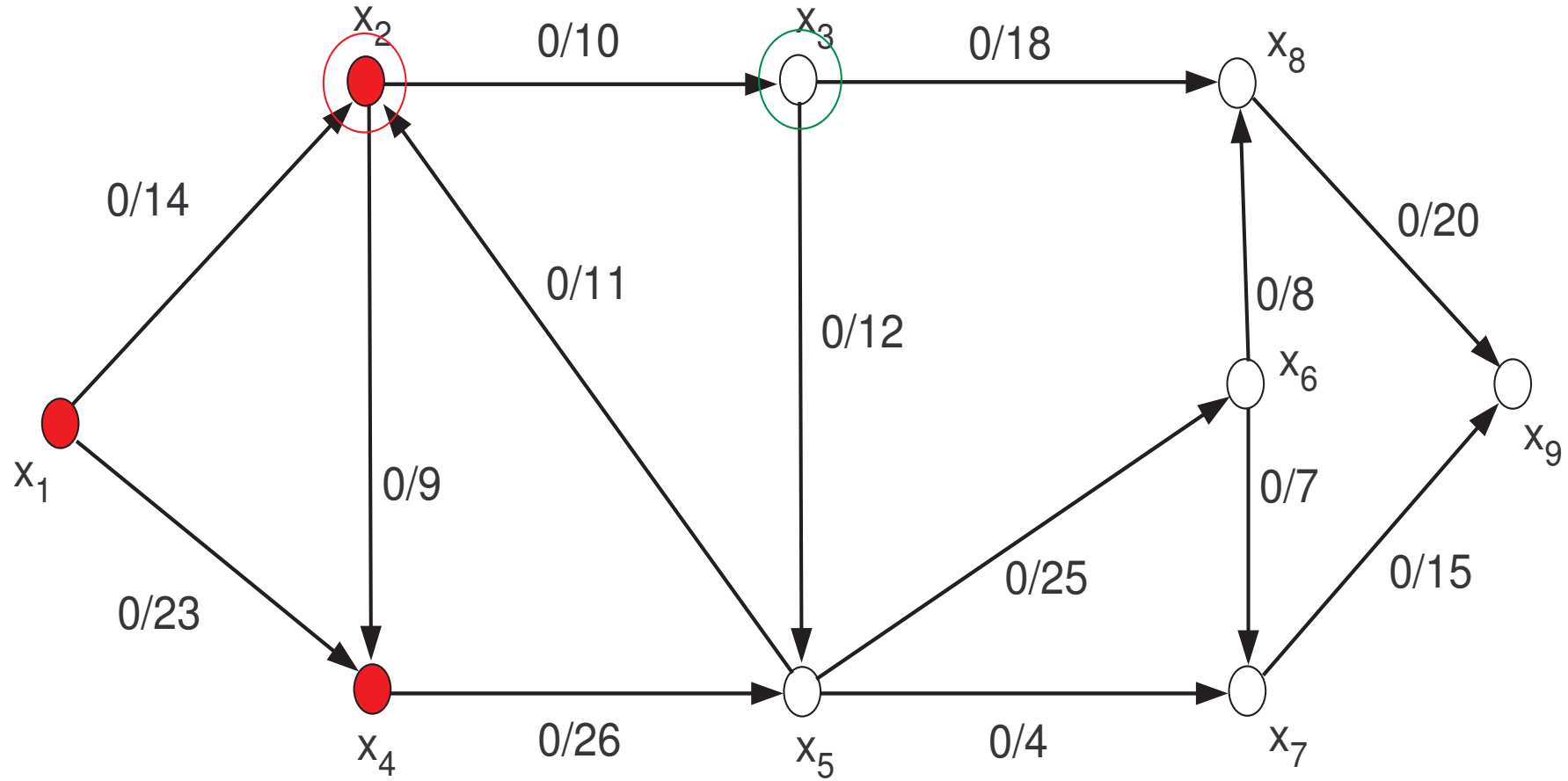
$$p(x_2) := +x_1, \delta(x_2) := 14$$

Ford-Fulkerson Algorithm



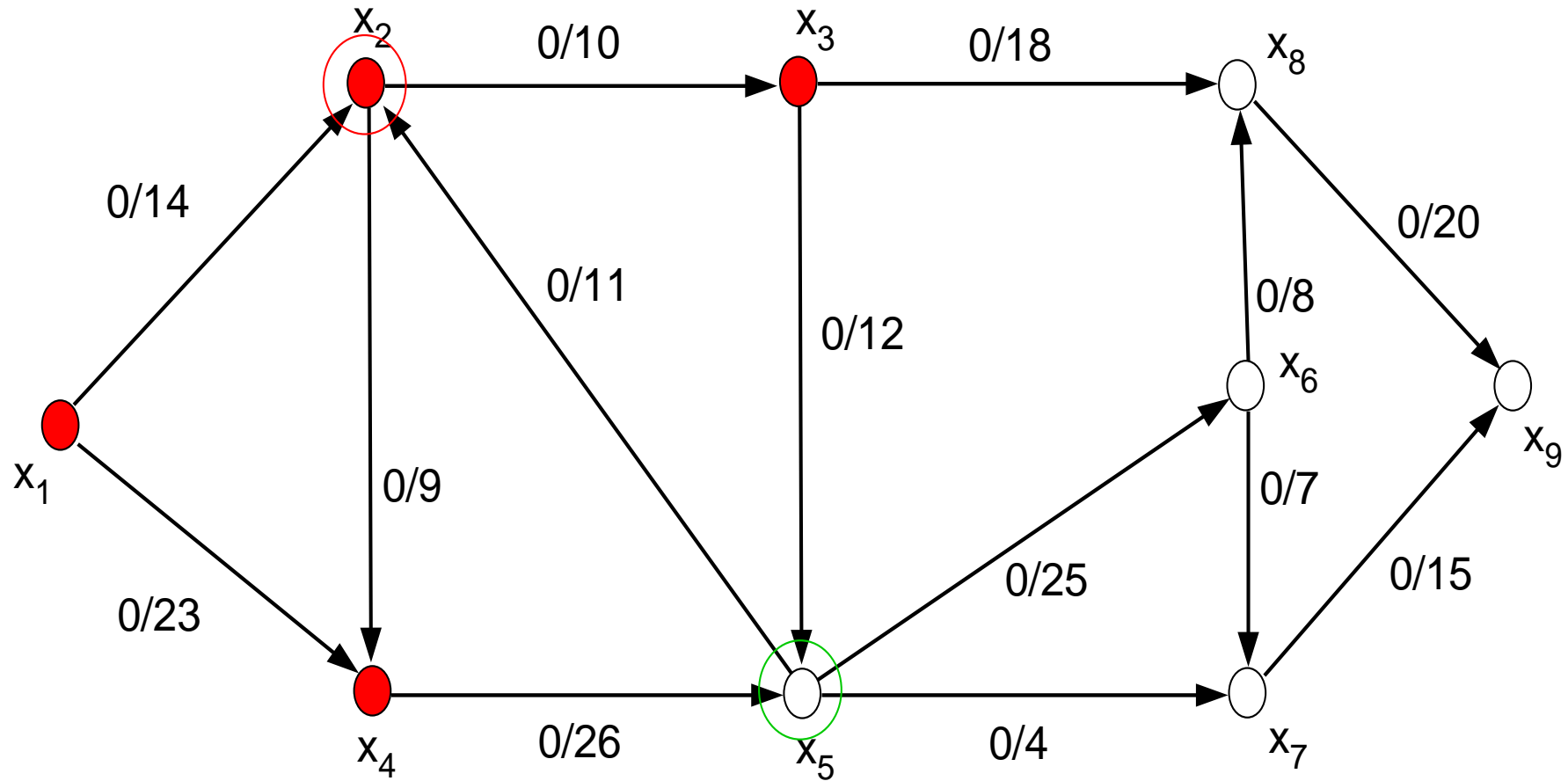
$$p(x_2) := +x_1, \delta(x_2) := 14, \quad p(x_4) := +x_1, \delta(x_4) := 23$$

Ford-Fulkerson Algorithm



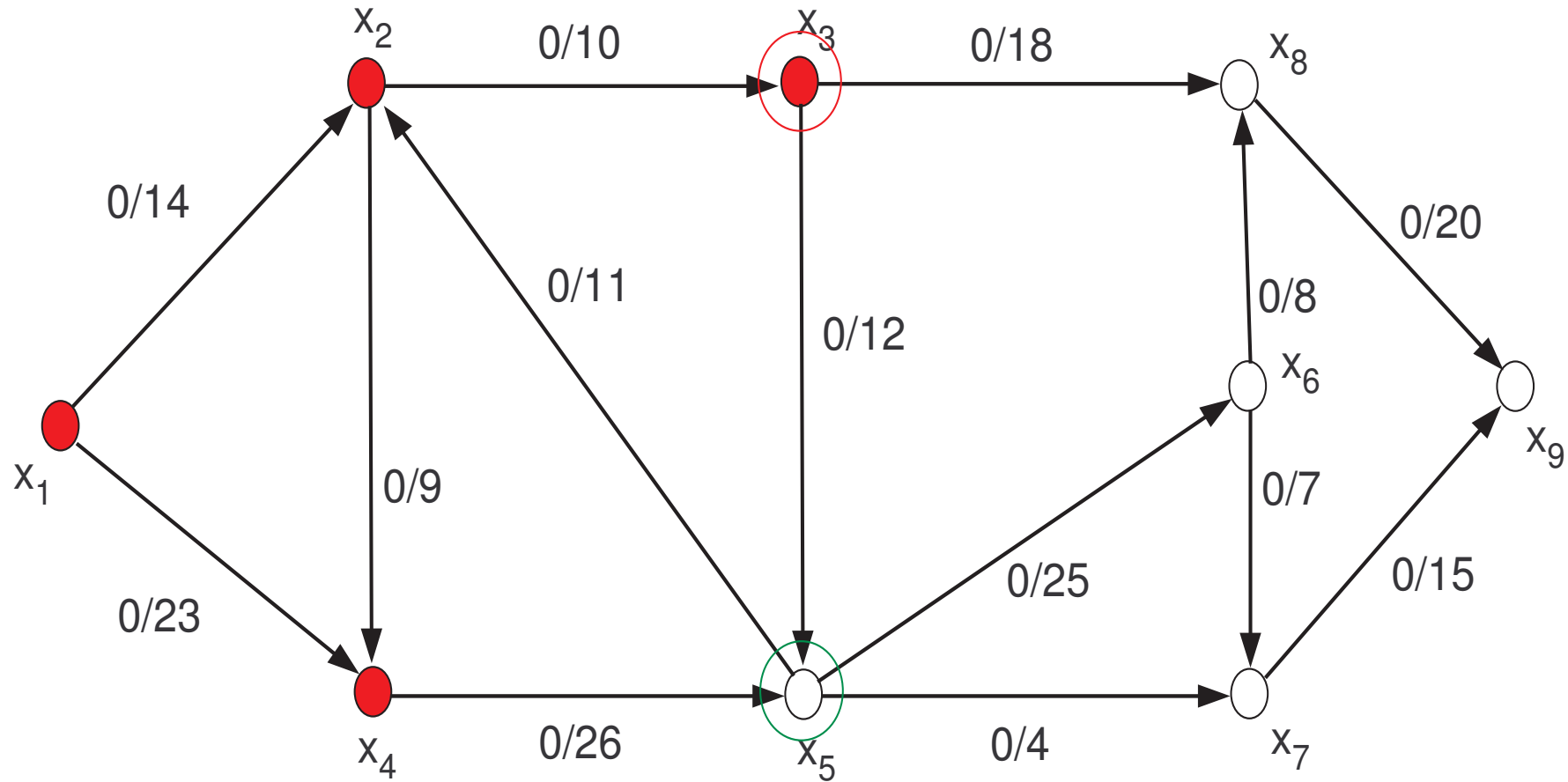
$$p(x_2) := +x_1, \delta(x_2) := 14, \quad p(x_4) := +x_1, \delta(x_4) := 23, \quad p(x_3) := +x_2, \delta(x_3) := 10$$

Ford-Fulkerson Algorithm



$$p(x_2) := +x_1, \delta(x_2) := 14, \quad p(x_4) := +x_1, \delta(x_4) := 23, \quad p(x_3) := +x_2, \delta(x_3) := 10$$

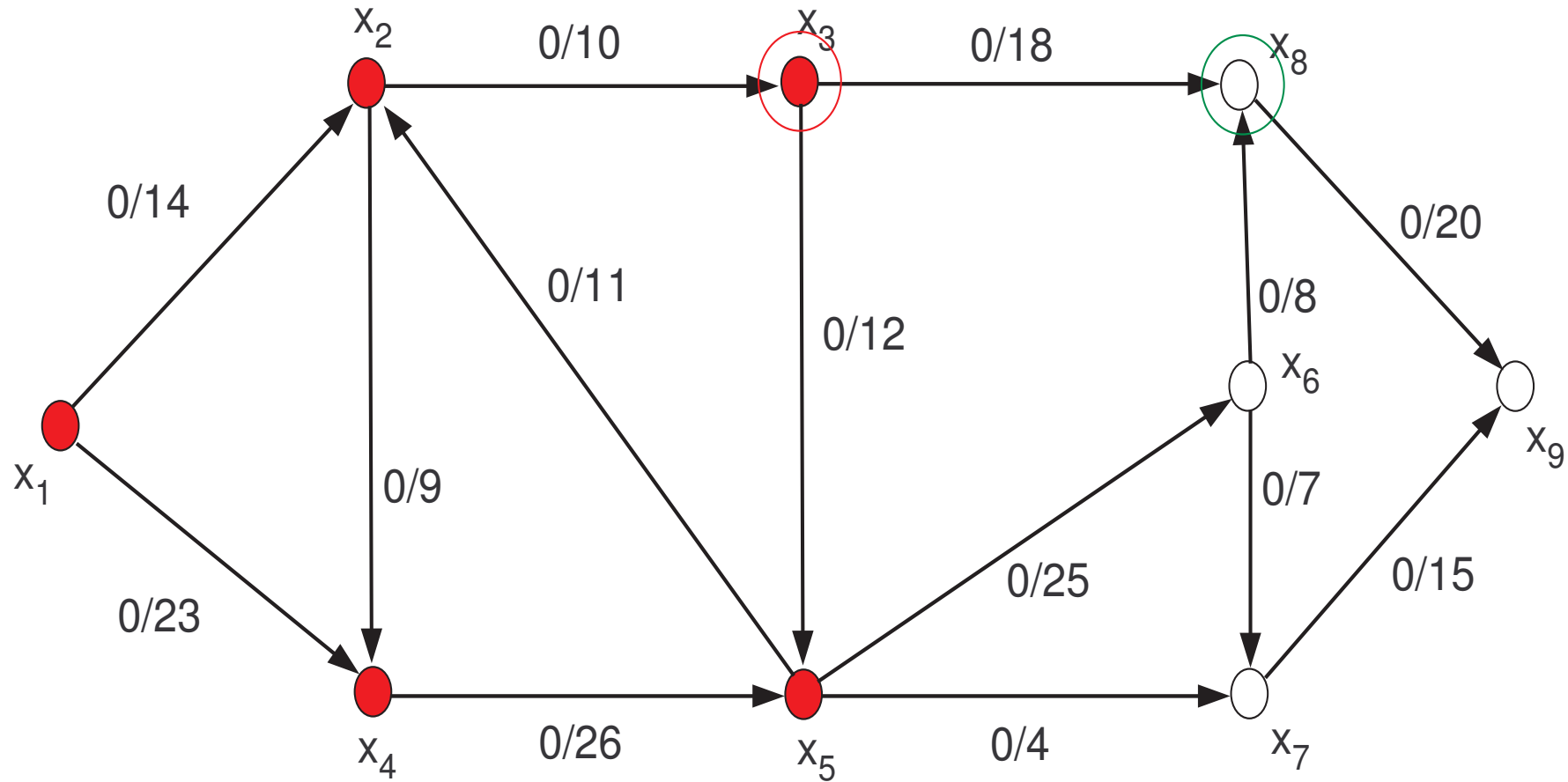
Ford-Fulkerson Algorithm



$$p(x_2) := +x_1, \delta(x_2) := 14, \quad p(x_4) := +x_1, \delta(x_4) := 23, \quad p(x_3) := +x_2, \delta(x_3) := 10,$$

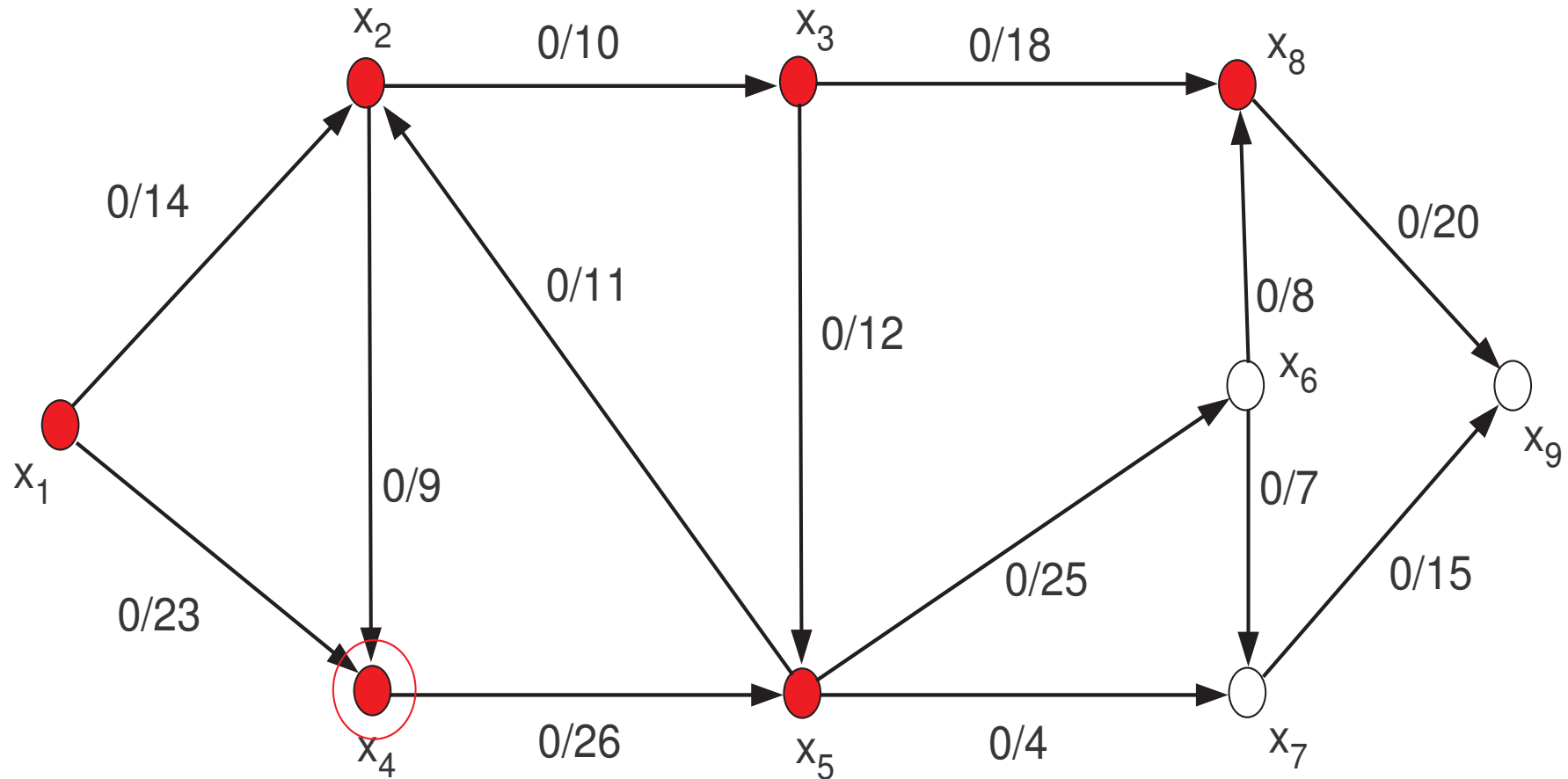
$$p(x_5) := +x_3, \delta(x_5) := 10$$

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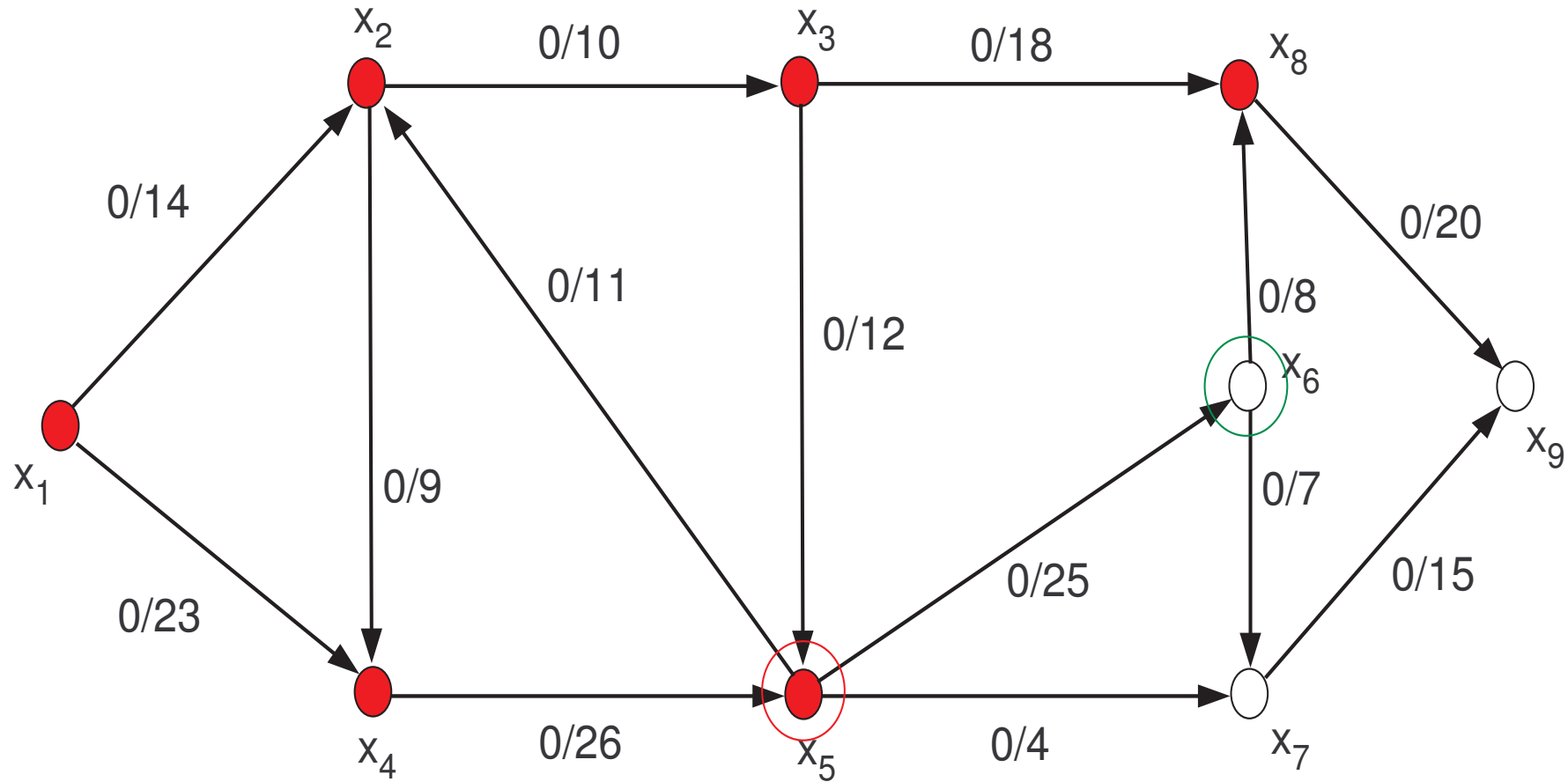
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 14, & p(x_4) &:= +x_1, \delta(x_4) := 23, & p(x_3) &:= +x_2, \delta(x_3) := 10, \\
 p(x_5) &:= +x_3, \delta(x_5) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10
 \end{aligned}$$

Ford-Fulkerson Algorithm



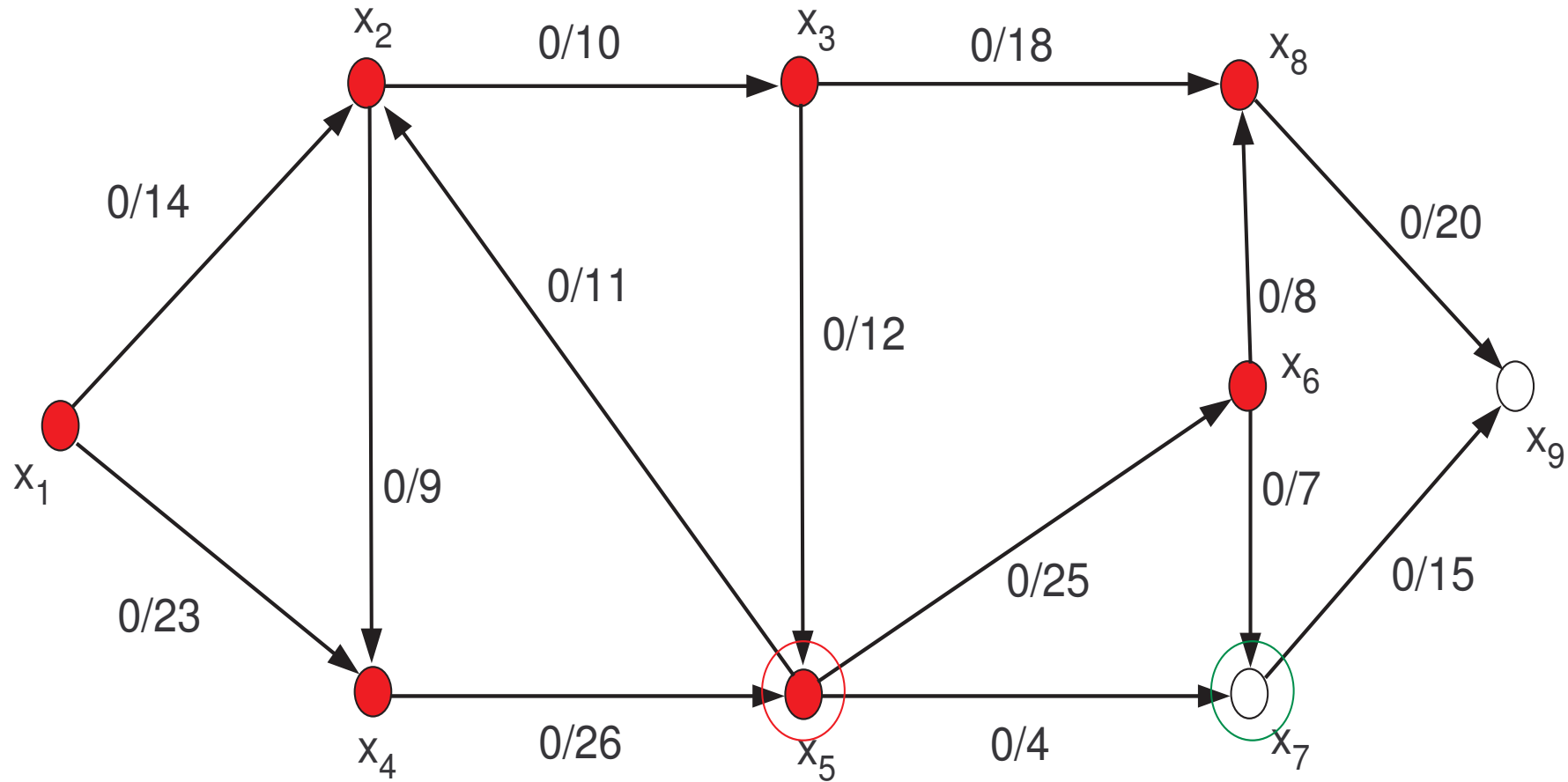
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 14, & p(x_4) &:= +x_1, \delta(x_4) := 23, & p(x_3) &:= +x_2, \delta(x_3) := 10, \\
 p(x_5) &:= +x_3, \delta(x_5) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10
 \end{aligned}$$

Ford-Fulkerson Algorithm



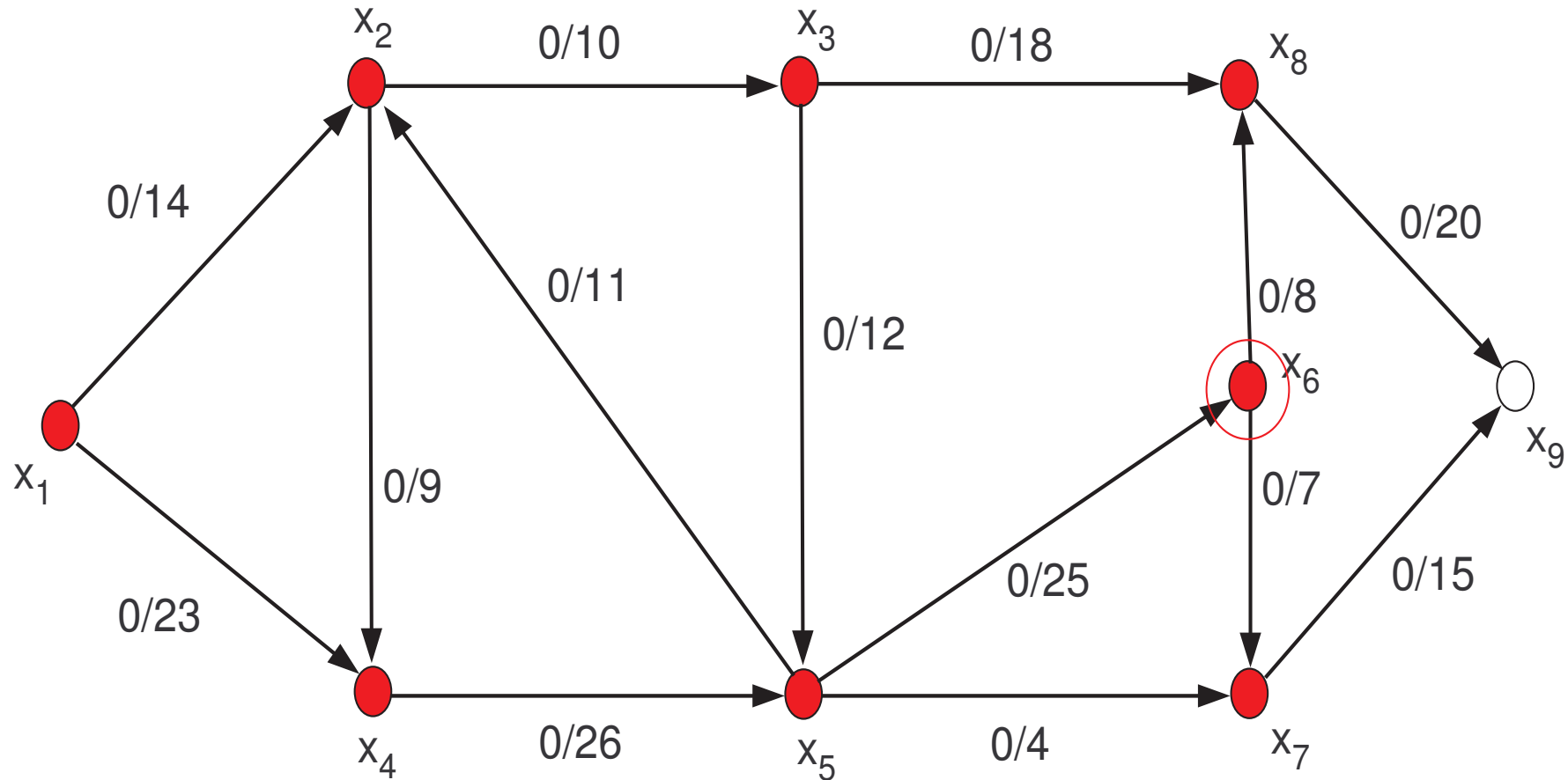
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 14, & p(x_4) &:= +x_1, \delta(x_4) := 23, & p(x_3) &:= +x_2, \delta(x_3) := 10, \\
 p(x_5) &:= +x_3, \delta(x_5) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10, & p(x_6) &:= +x_5, \delta(x_6) := 10
 \end{aligned}$$

Ford-Fulkerson Algorithm



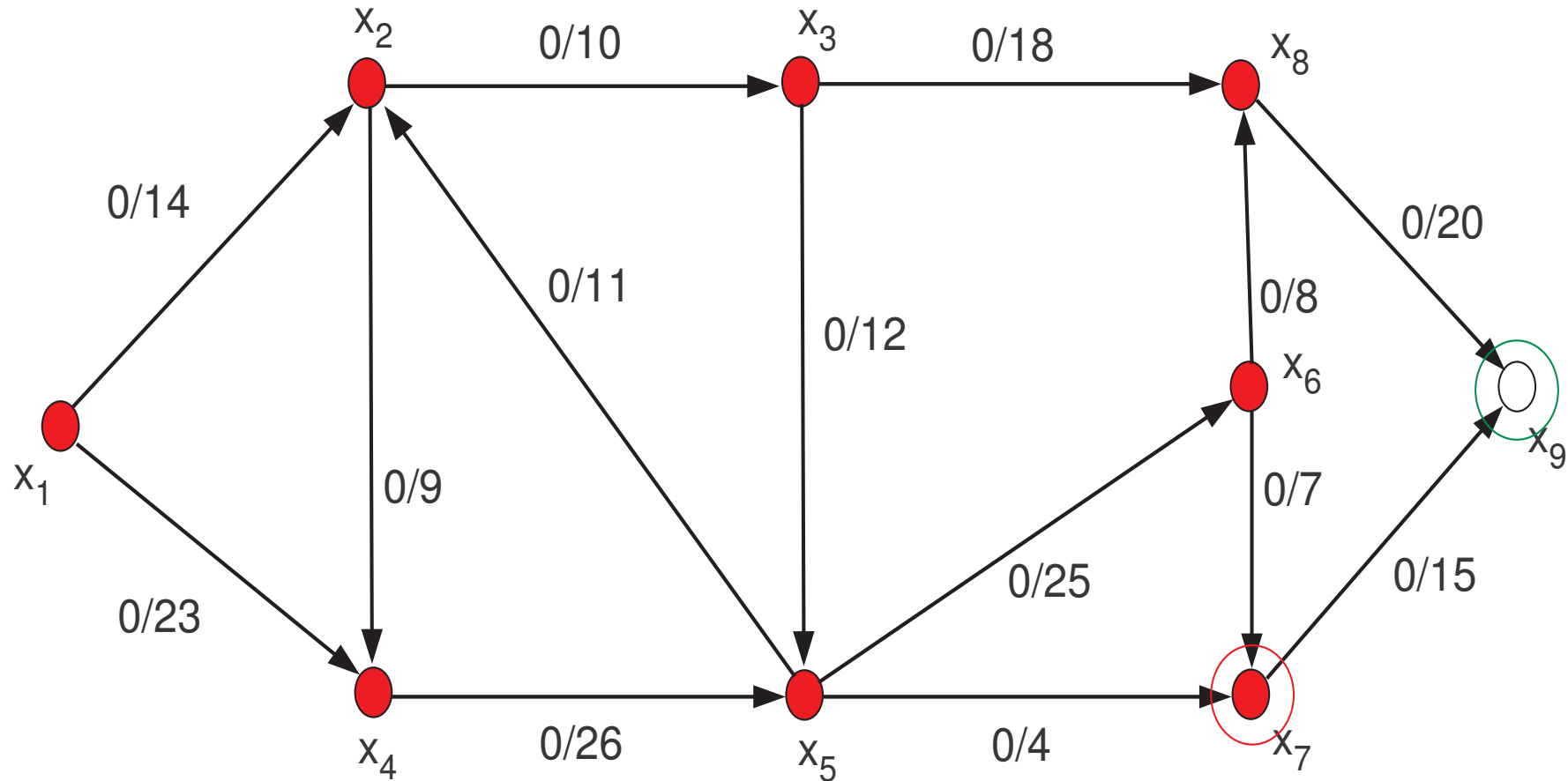
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 14, & p(x_4) &:= +x_1, \delta(x_4) := 23, & p(x_3) &:= +x_2, \delta(x_3) := 10, \\
 p(x_5) &:= +x_3, \delta(x_5) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10, & p(x_6) &:= +x_5, \delta(x_6) := 10, \\
 p(x_7) &:= +x_5, \delta(x_7) := 4
 \end{aligned}$$

Ford-Fulkerson Algorithm



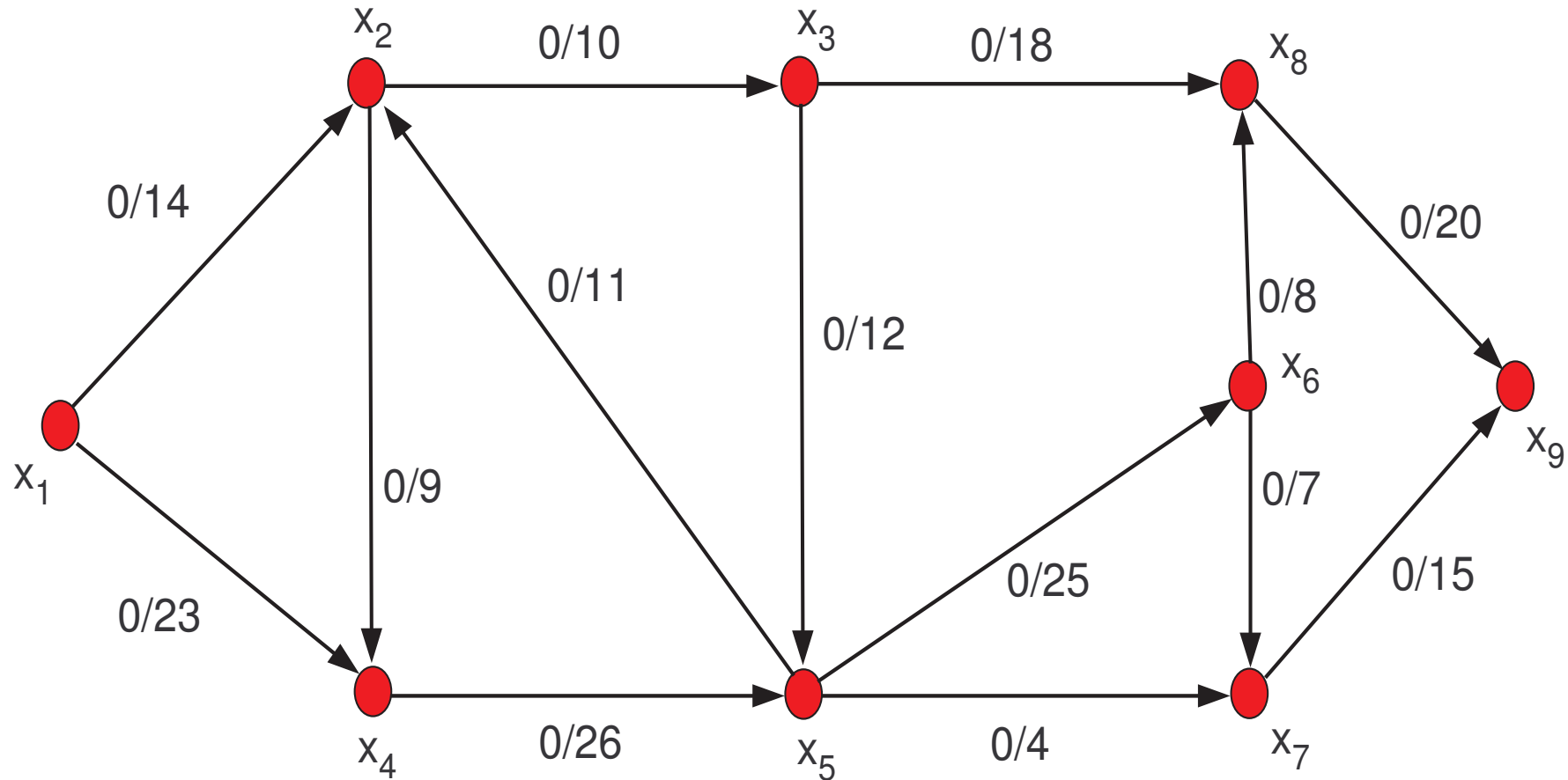
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 14, & p(x_4) &:= +x_1, \delta(x_4) := 23, & p(x_3) &:= +x_2, \delta(x_3) := 10, \\
 p(x_5) &:= +x_3, \delta(x_5) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10, & p(x_6) &:= +x_5, \delta(x_6) := 10, \\
 p(x_7) &:= +x_5, \delta(x_7) := 4
 \end{aligned}$$

Ford-Fulkerson Algorithm



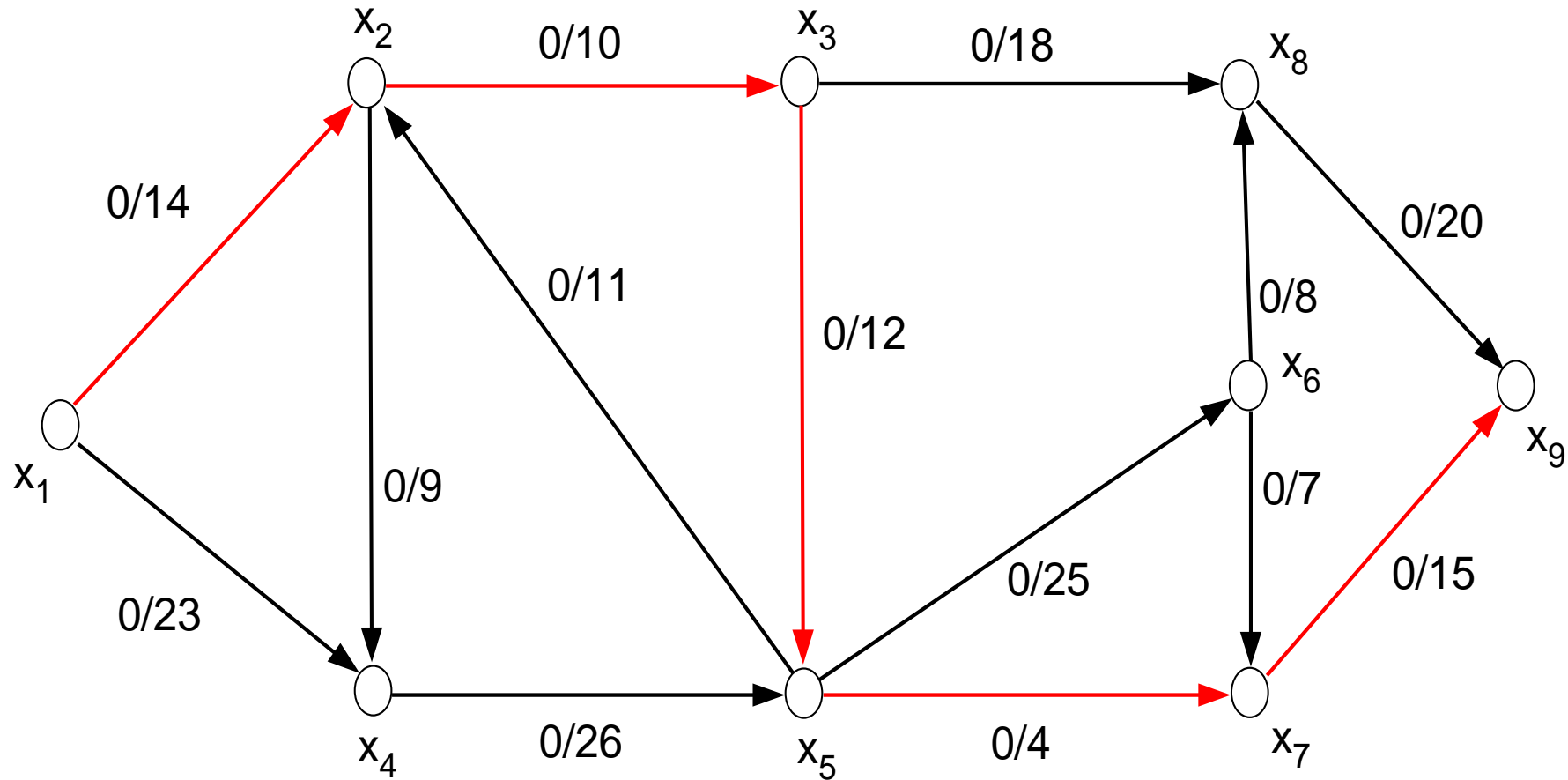
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 14, & p(x_4) &:= +x_1, \delta(x_4) := 23, & p(x_3) &:= +x_2, \delta(x_3) := 10, \\
 p(x_5) &:= +x_3, \delta(x_5) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10, & p(x_6) &:= +x_5, \delta(x_6) := 10, \\
 p(x_7) &:= +x_5, \delta(x_7) := 4, & p(x_9) &:= +x_7, \delta(x_9) := 4
 \end{aligned}$$

Ford-Fulkerson Algorithm



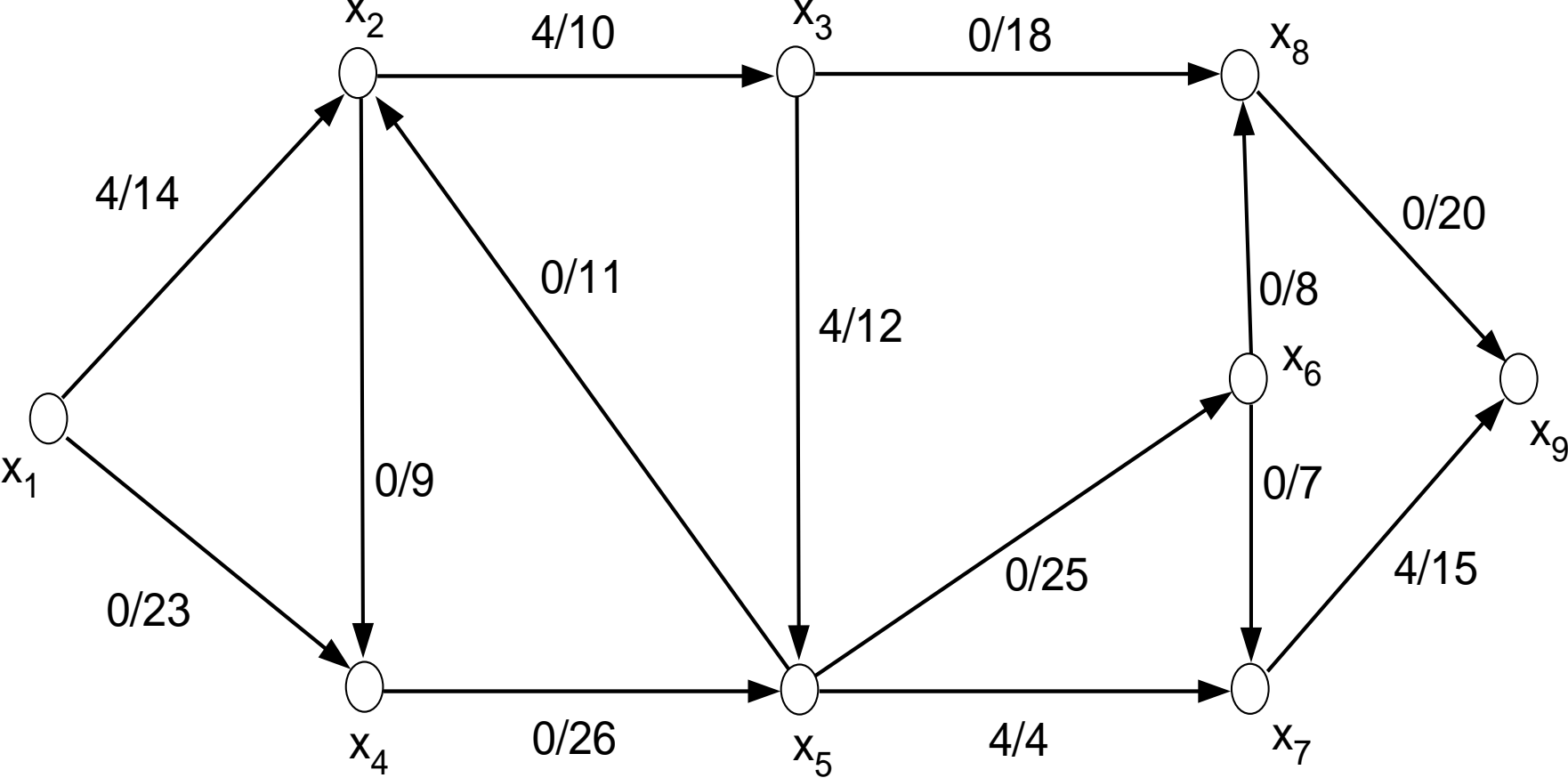
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 14, & p(x_4) &:= +x_1, \delta(x_4) := 23, & p(x_3) &:= +x_2, \delta(x_3) := 10, \\
 p(x_5) &:= +x_3, \delta(x_5) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10, & p(x_6) &:= +x_5, \delta(x_6) := 10, \\
 p(x_7) &:= +x_5, \delta(x_7) := 4, & p(x_9) &:= +x_7, \delta(x_9) := 4
 \end{aligned}$$

Ford-Fulkerson Algorithm

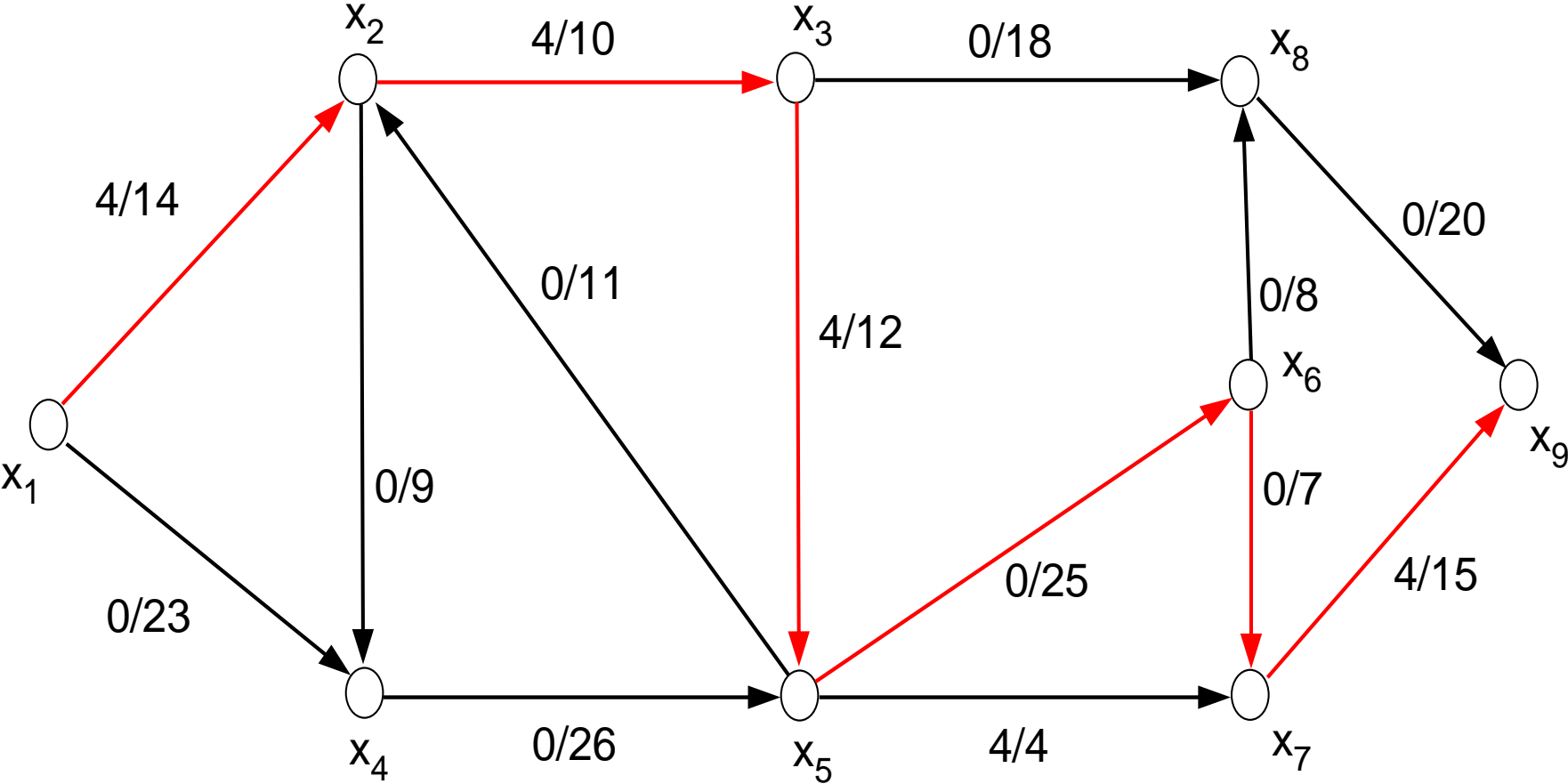


$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 14, & p(x_4) &:= +x_1, \delta(x_4) := 23, & p(x_3) &:= +x_2, \delta(x_3) := 10, \\
 p(x_5) &:= +x_3, \delta(x_5) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10, & p(x_6) &:= +x_5, \delta(x_6) := 10, \\
 p(x_7) &:= +x_5, \delta(x_7) := 4, & p(x_9) &:= +x_7, \delta(x_9) := 4
 \end{aligned}$$

Ford-Fulkerson Algorithm



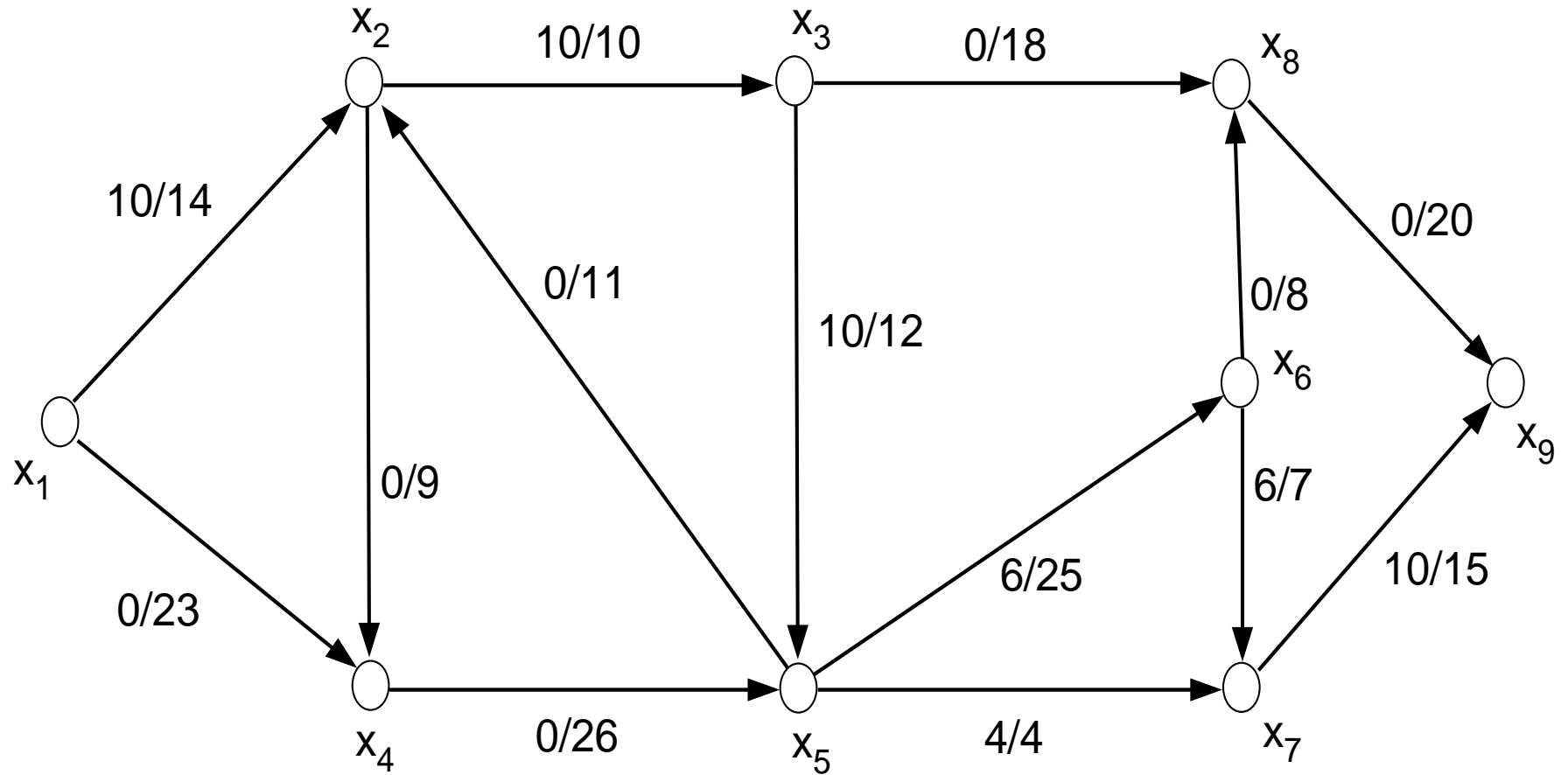
Ford-Fulkerson Algorithm



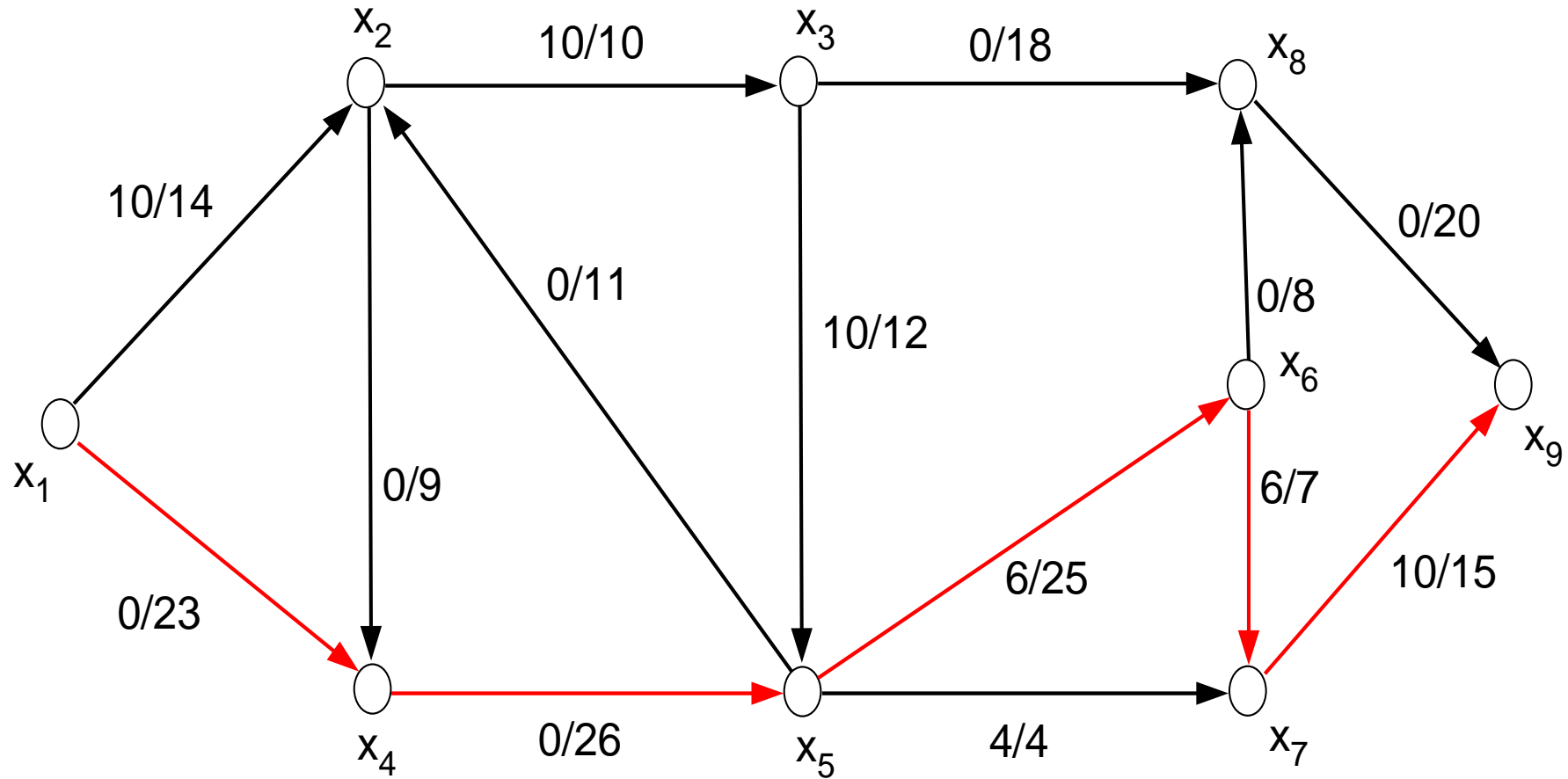
$x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_5 \rightarrow x_6 \rightarrow x_7 \rightarrow x_9$

$\phi(e) = \delta(x_9) = 6$

Ford-Fulkerson Algorithm



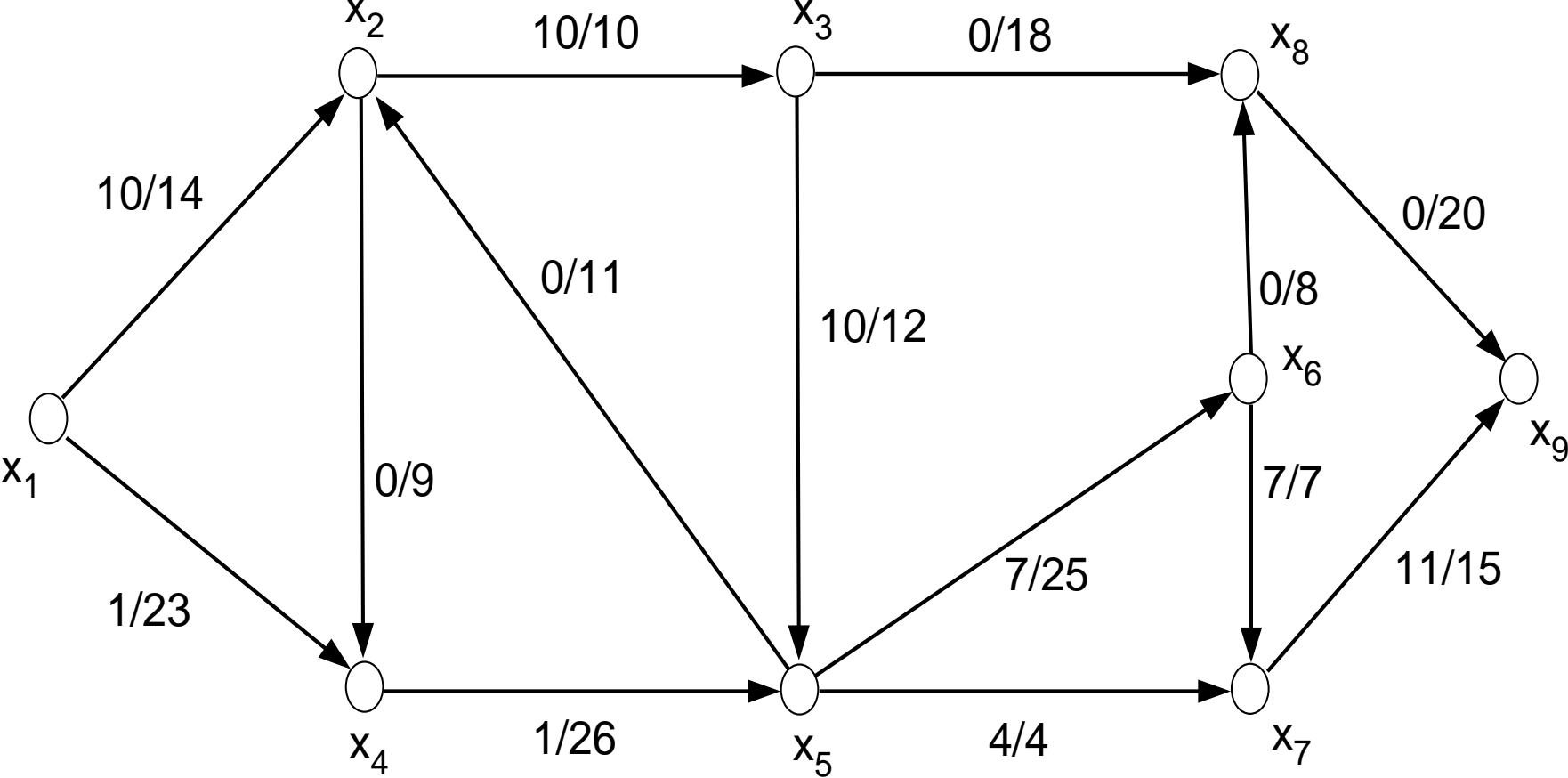
Ford-Fulkerson Algorithm



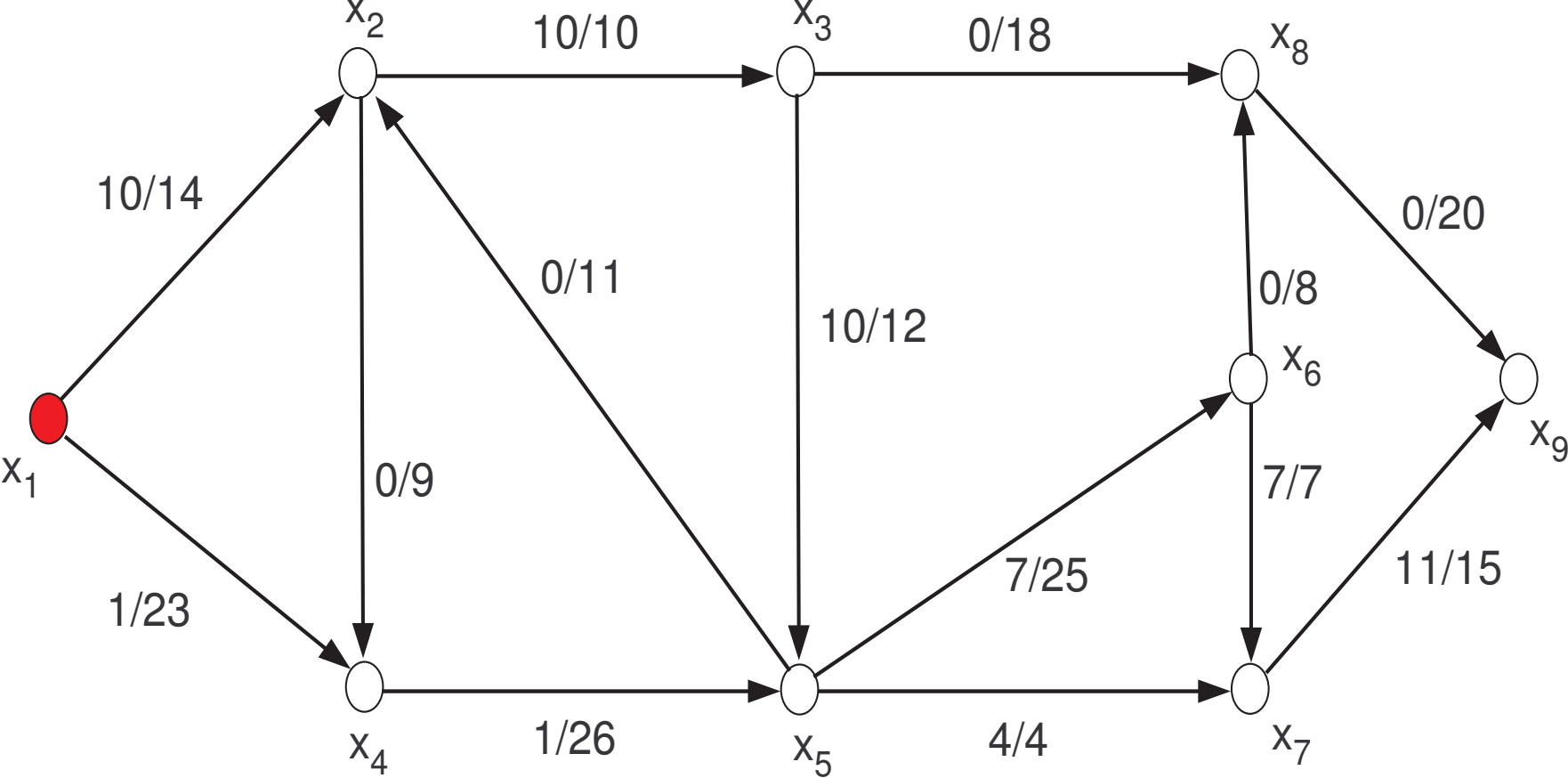
$x_1 \longrightarrow x_4 \longrightarrow x_5 \longrightarrow x_6 \longrightarrow x_7 \longrightarrow x_9$

$$\phi(e) = \delta(x_9) = 1$$

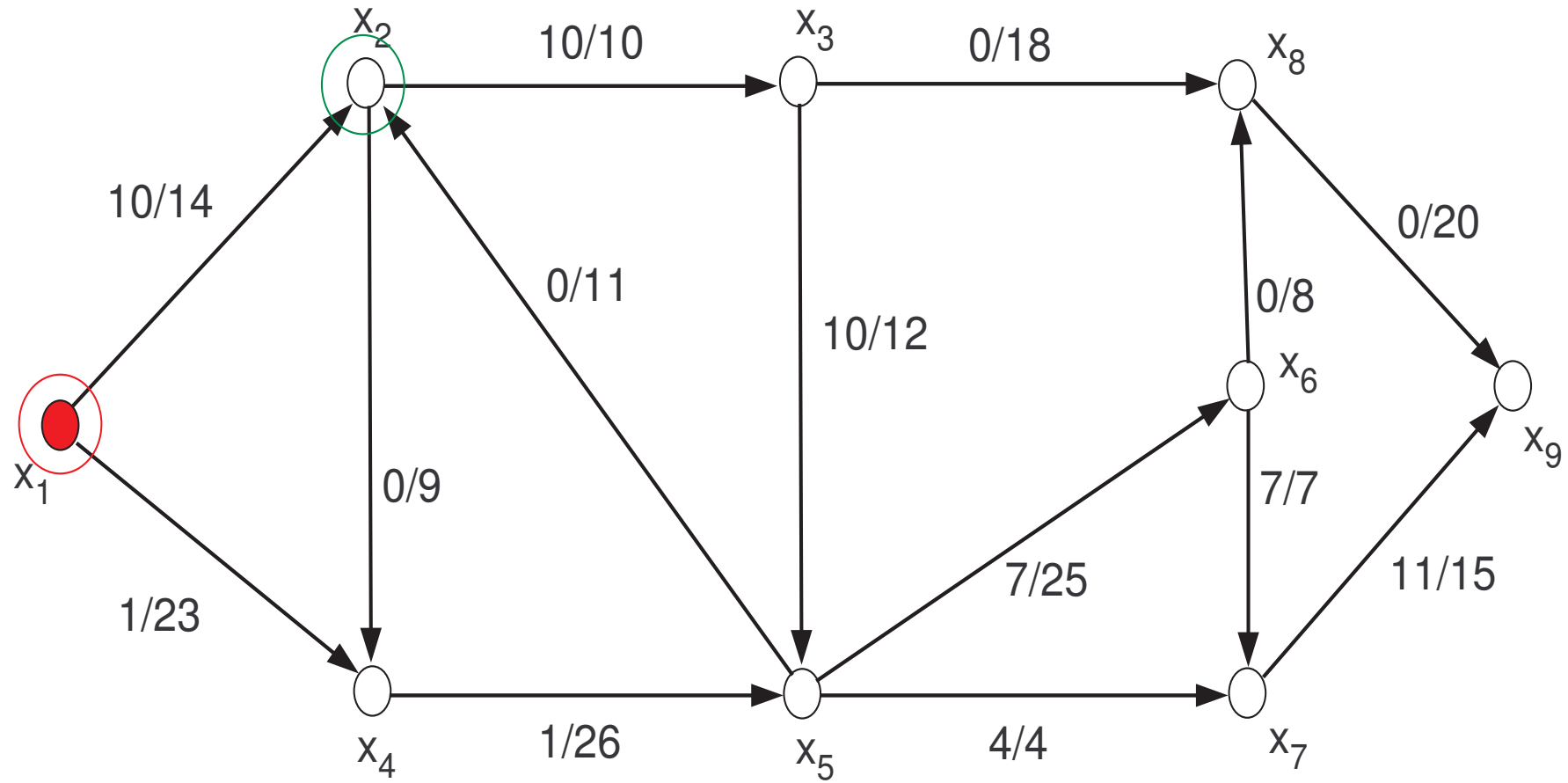
Ford-Fulkerson Algorithm



Ford-Fulkerson Algorithm

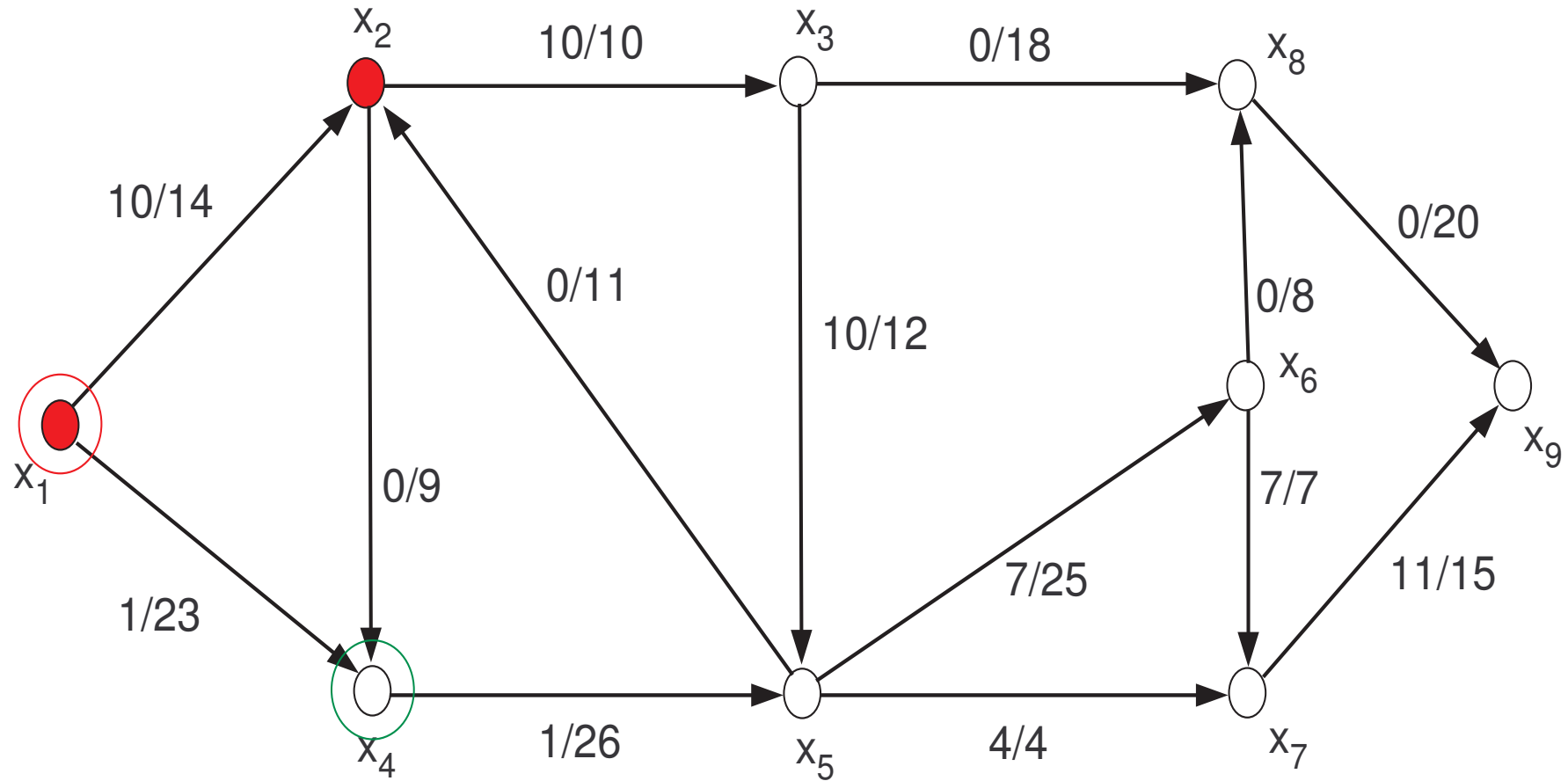


Ford-Fulkerson Algorithm



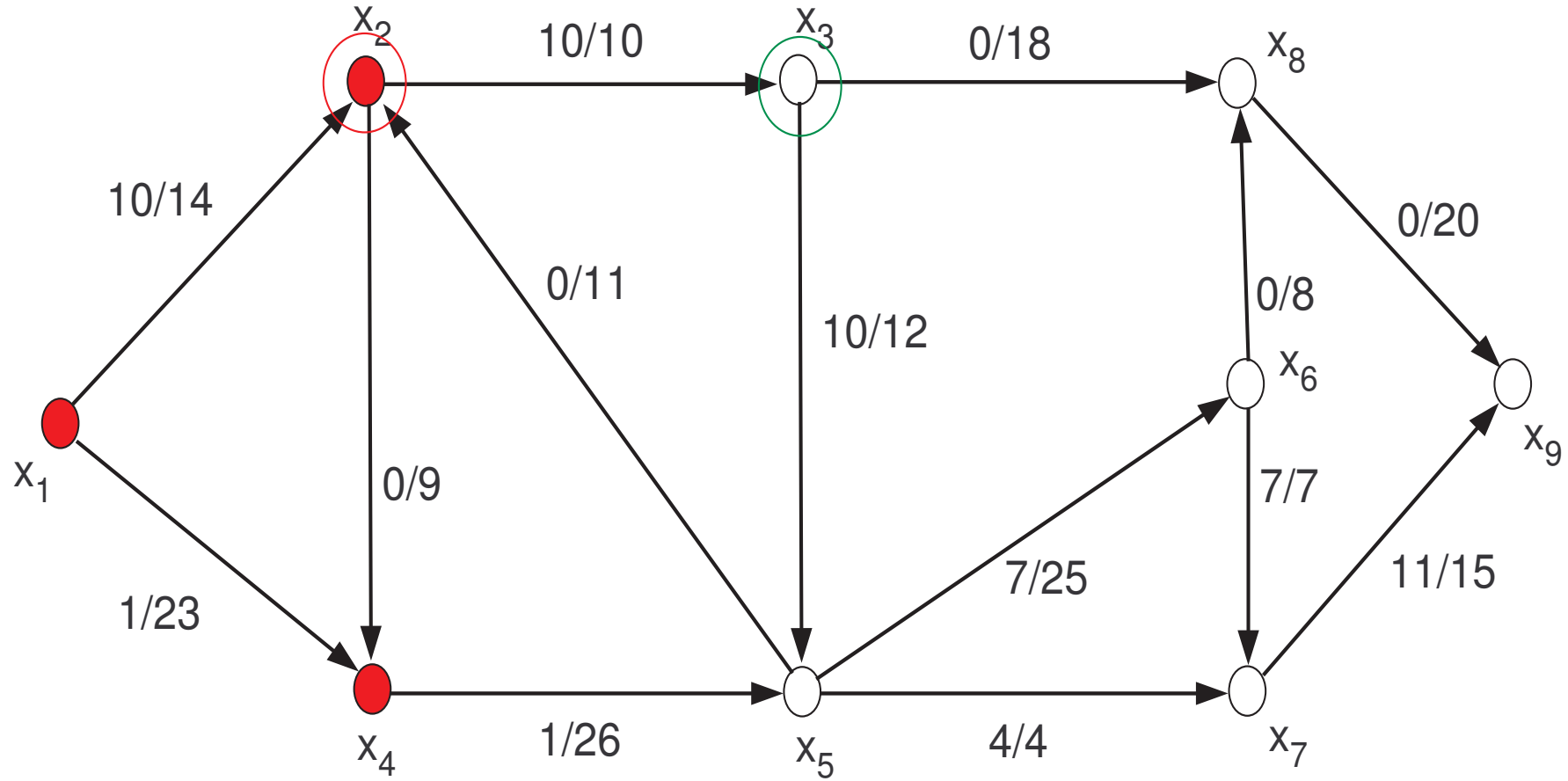
$$p(x_2) := +x_1, \delta(x_2) := 4$$

Ford-Fulkerson Algorithm



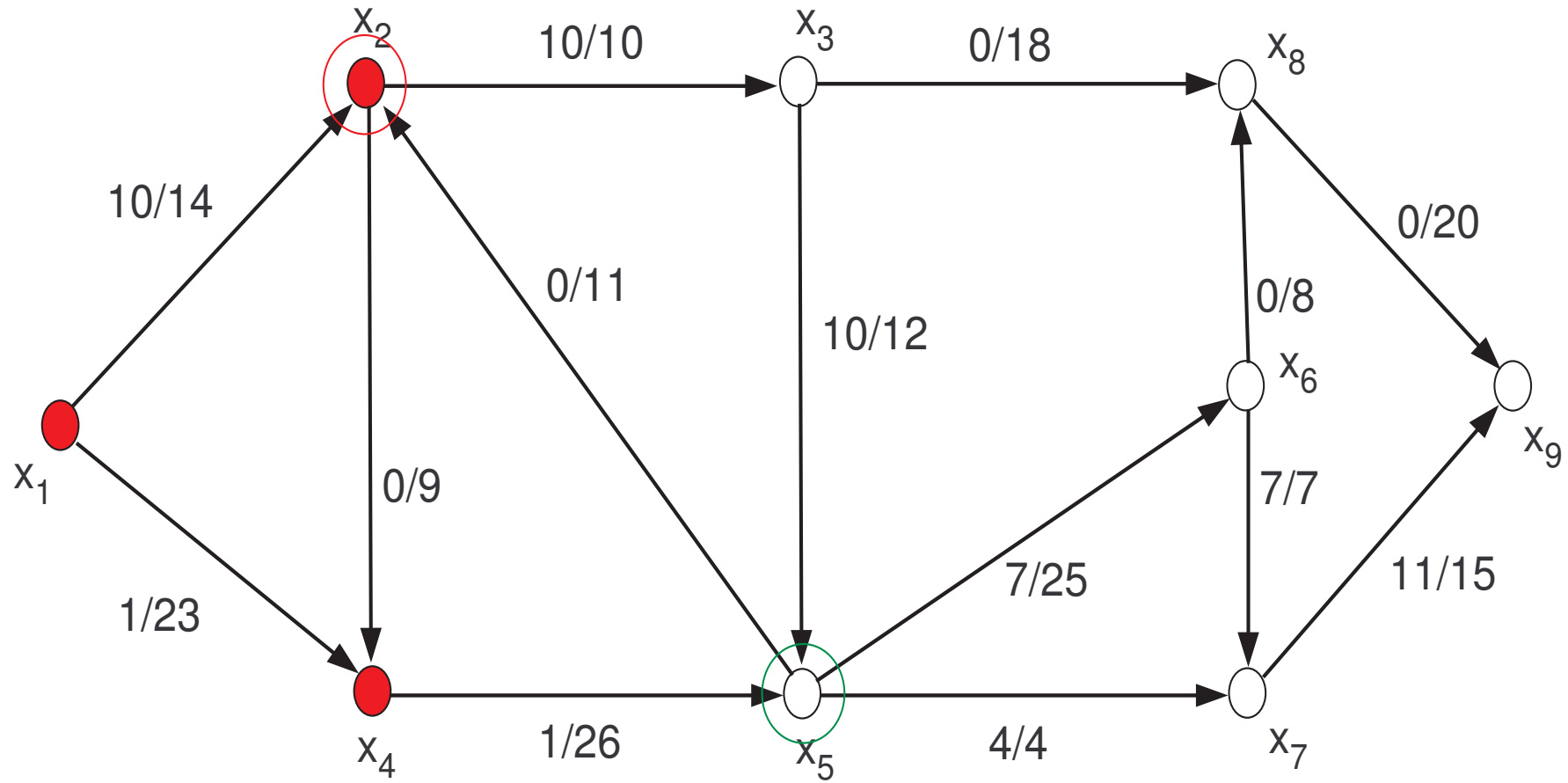
$$p(x_2) := +x_1, \delta(x_2) := 4, \quad p(x_4) := +x_1, \delta(x_4) := 22$$

Ford-Fulkerson Algorithm



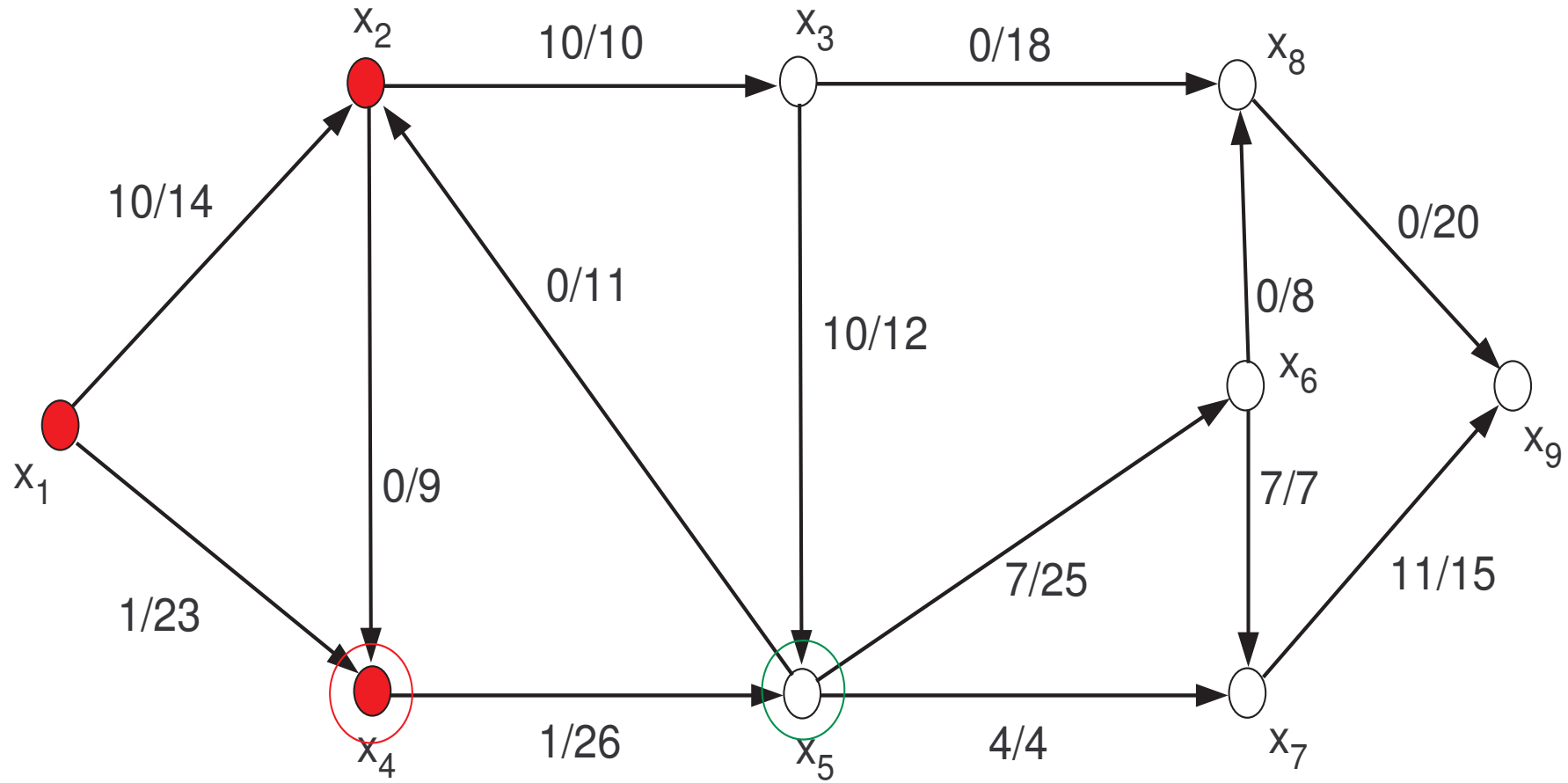
$$p(x_2) := +x_1, \delta(x_2) := 4, \quad p(x_4) := +x_1, \delta(x_4) := 22$$

Ford-Fulkerson Algorithm



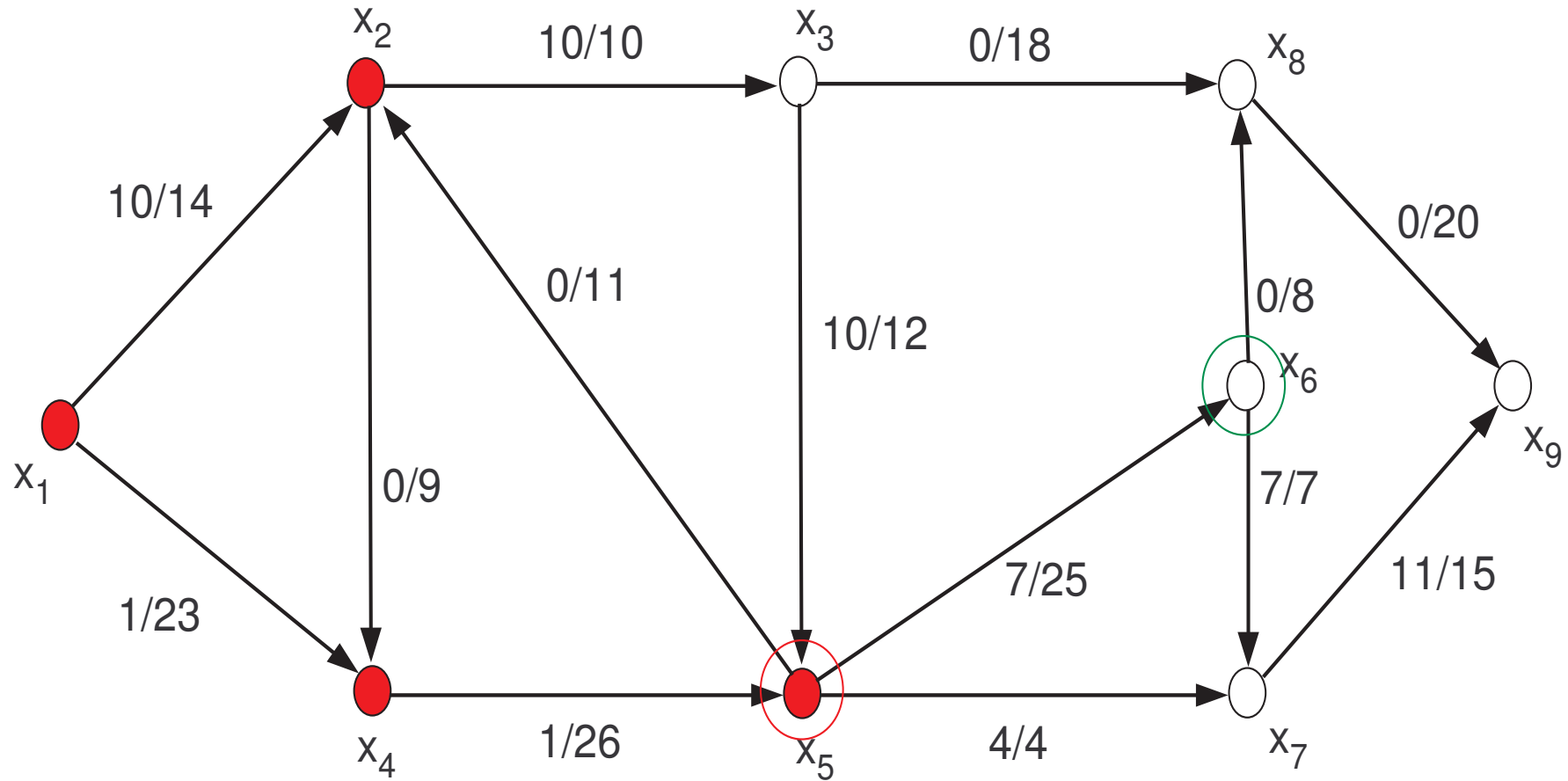
$$p(x_2) := +x_1, \delta(x_2) := 4, \quad p(x_4) := +x_1, \delta(x_4) := 22$$

Ford-Fulkerson Algorithm



$$p(x_2) := +x_1, \delta(x_2) := 4, \quad p(x_4) := +x_1, \delta(x_4) := 22, \quad p(x_5) := +x_4, \delta(x_5) := 22$$

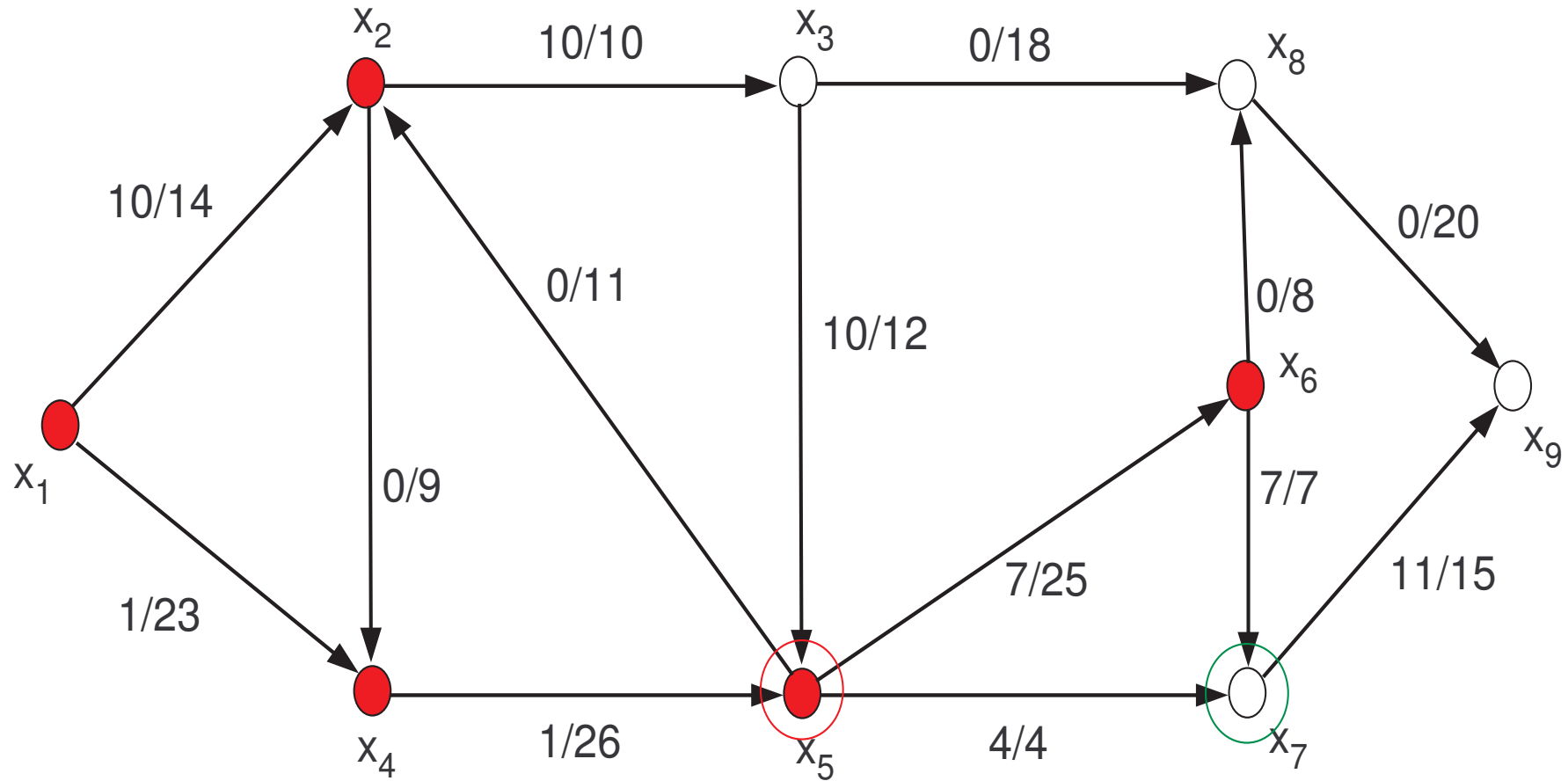
Ford-Fulkerson Algorithm



$$p(x_2) := +x_1, \delta(x_2) := 4, \quad p(x_4) := +x_1, \delta(x_4) := 22, \quad p(x_5) := +x_4, \delta(x_5) := 22,$$

$$p(x_6) := +x_5, \delta(x_6) := 18$$

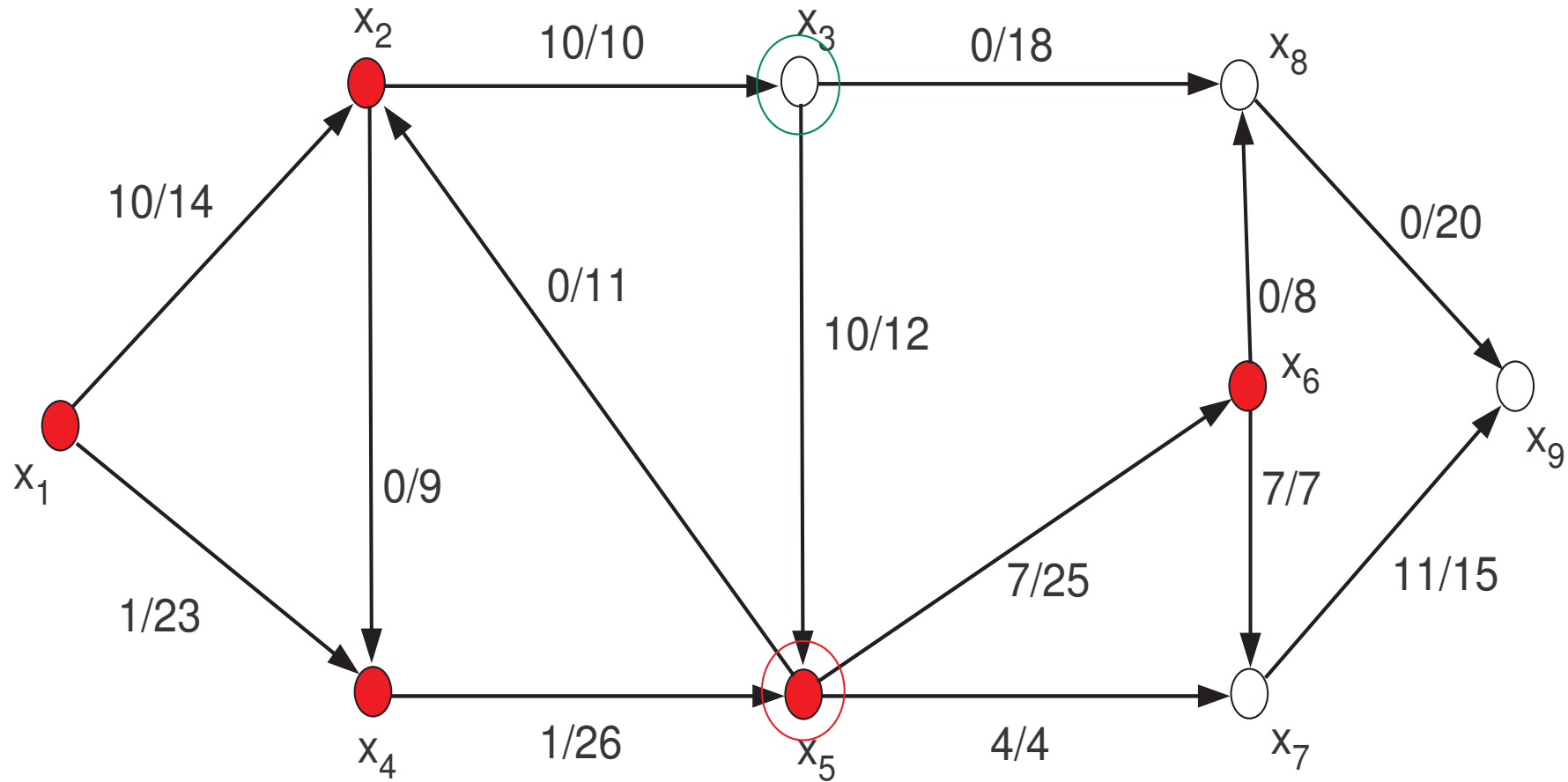
Ford-Fulkerson Algorithm



$$p(x_2) := +x_1, \delta(x_2) := 4, \quad p(x_4) := +x_1, \delta(x_4) := 22, \quad p(x_5) := +x_4, \delta(x_5) := 22,$$

$$p(x_6) := +x_5, \delta(x_6) := 18$$

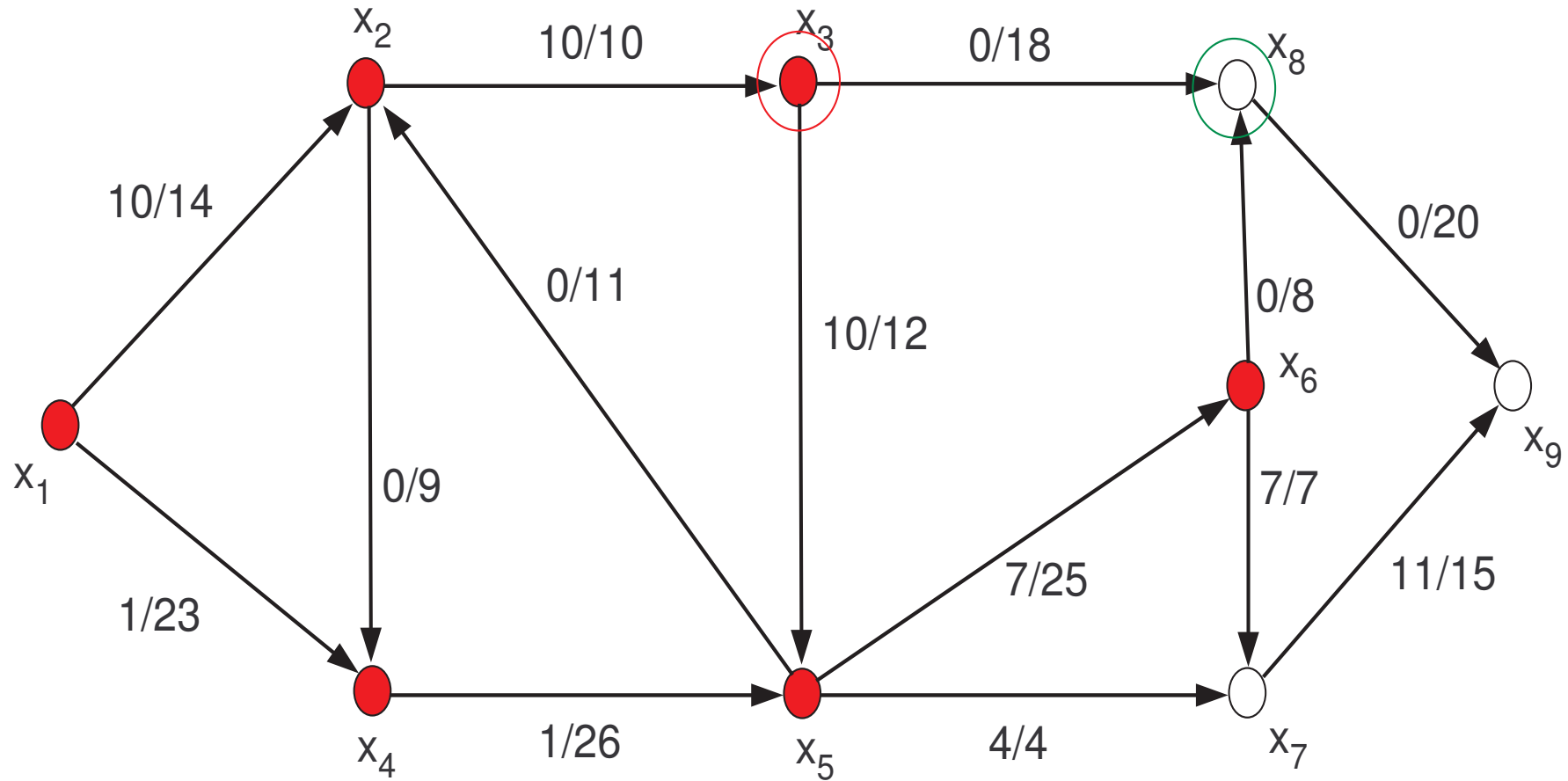
Ford-Fulkerson Algorithm



$$p(x_2) := +x_1, \delta(x_2) := 4, \quad p(x_4) := +x_1, \delta(x_4) := 22, \quad p(x_5) := +x_4, \delta(x_5) := 22,$$

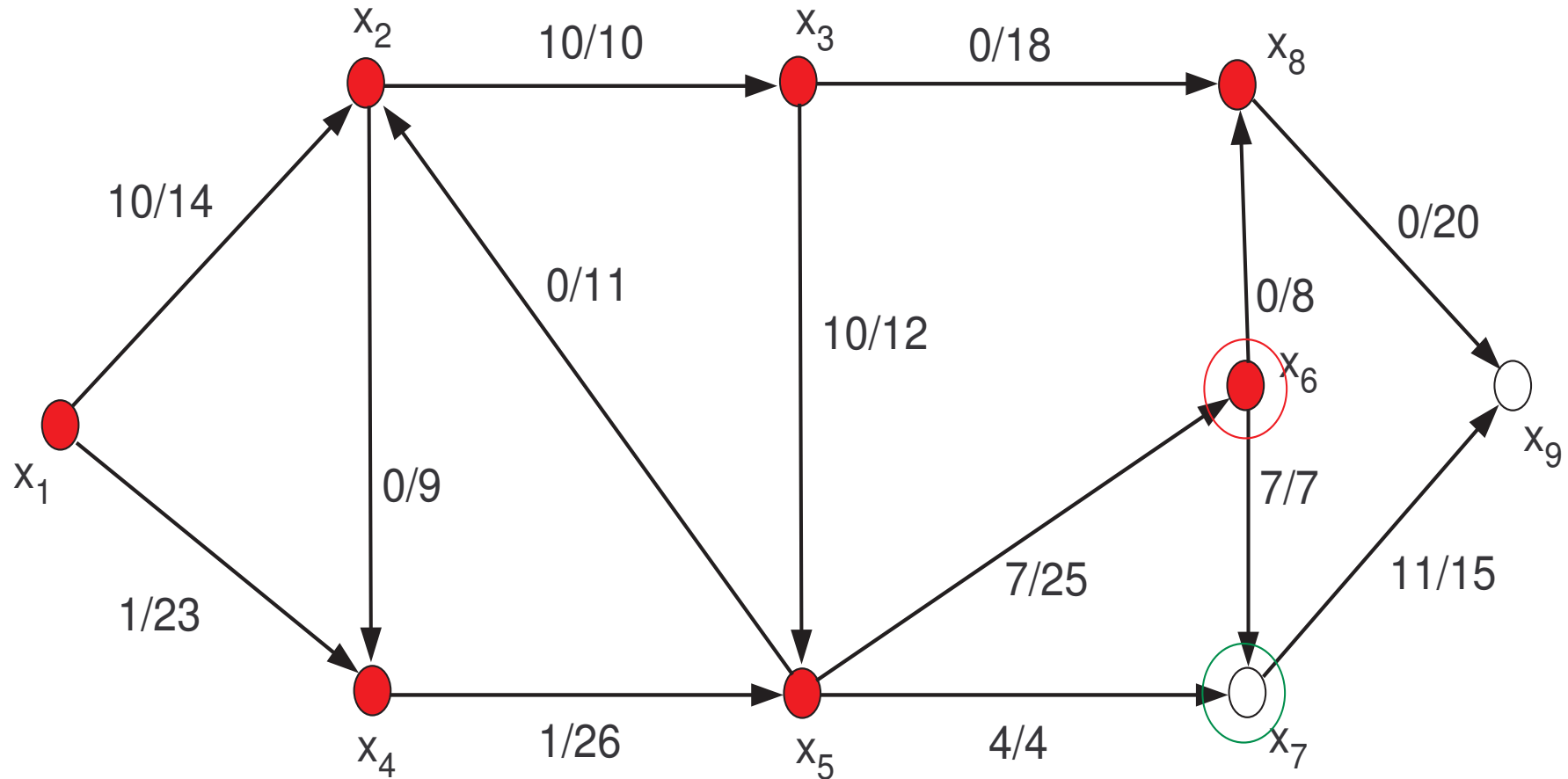
$$p(x_6) := +x_5, \delta(x_6) := 18, \quad p(x_3) := -x_5, \delta(x_3) := 10$$

Ford-Fulkerson Algorithm



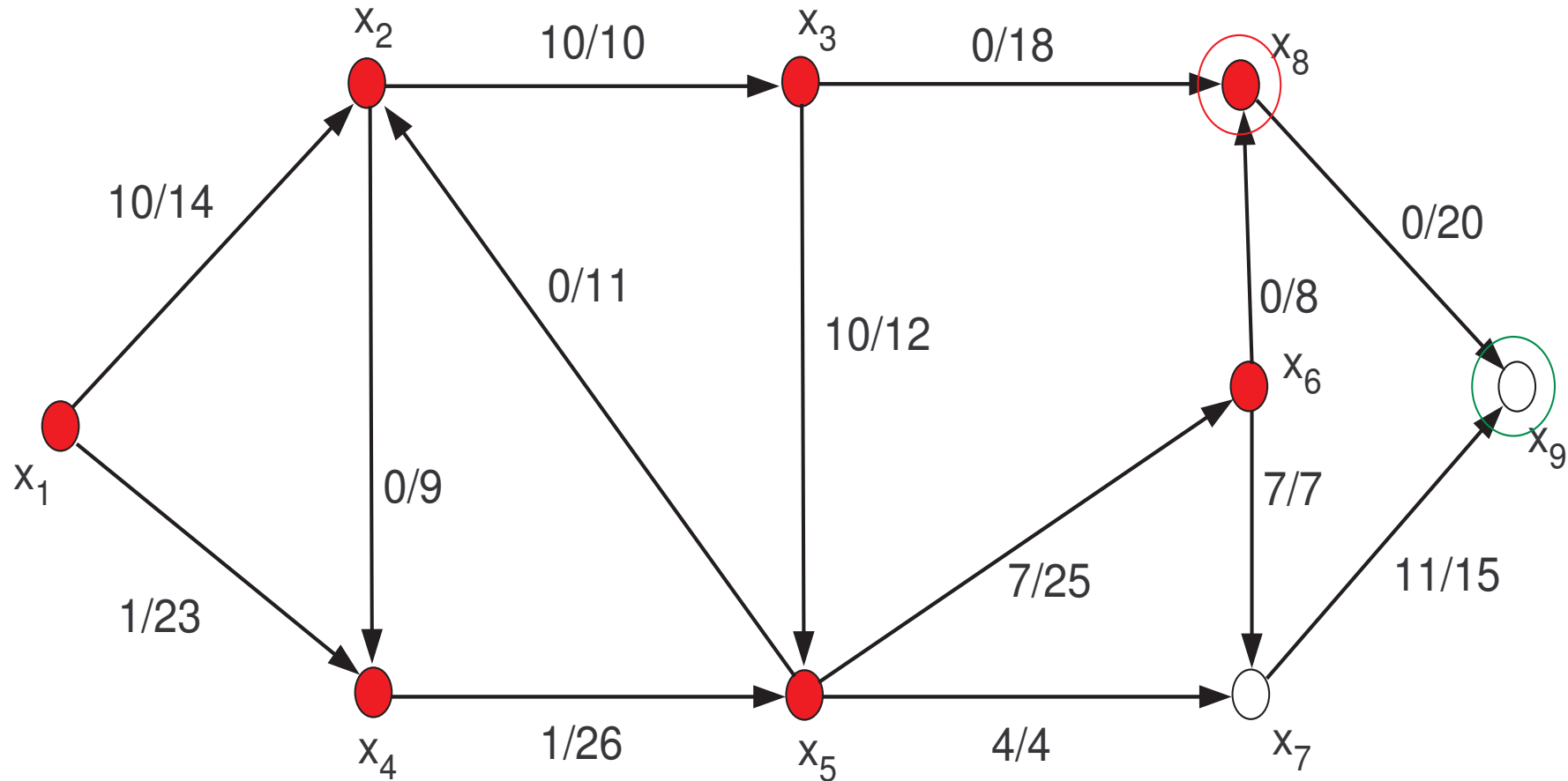
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 4, & p(x_4) &:= +x_1, \delta(x_4) := 22, & p(x_5) &:= +x_4, \delta(x_5) := 22, \\
 p(x_6) &:= +x_5, \delta(x_6) := 18, & p(x_3) &:= -x_5, \delta(x_3) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10
 \end{aligned}$$

Ford-Fulkerson Algorithm



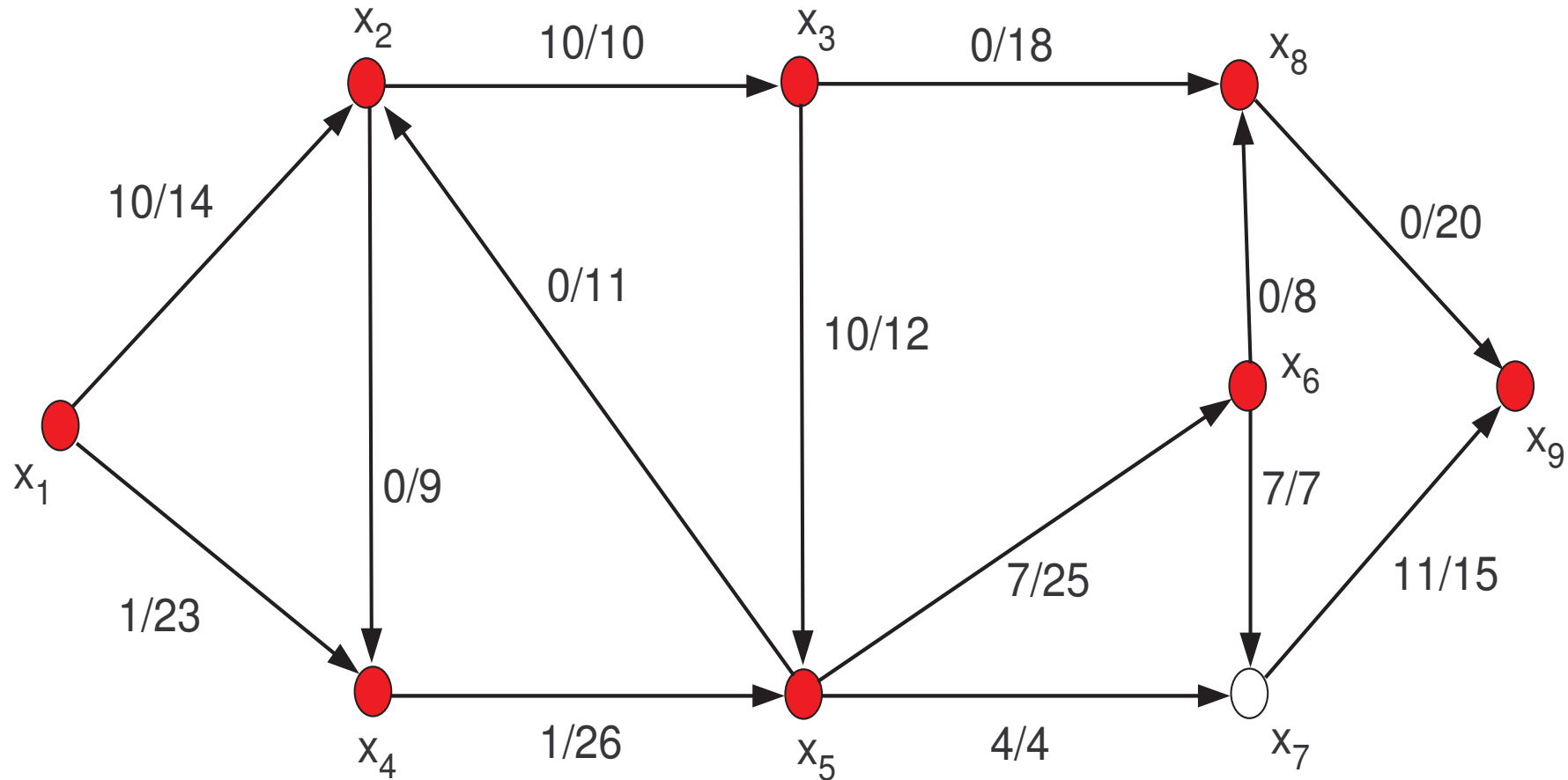
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 4, & p(x_4) &:= +x_1, \delta(x_4) := 22, & p(x_5) &:= +x_4, \delta(x_5) := 22, \\
 p(x_6) &:= +x_5, \delta(x_6) := 18, & p(x_3) &:= -x_5, \delta(x_3) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10
 \end{aligned}$$

Ford-Fulkerson Algorithm



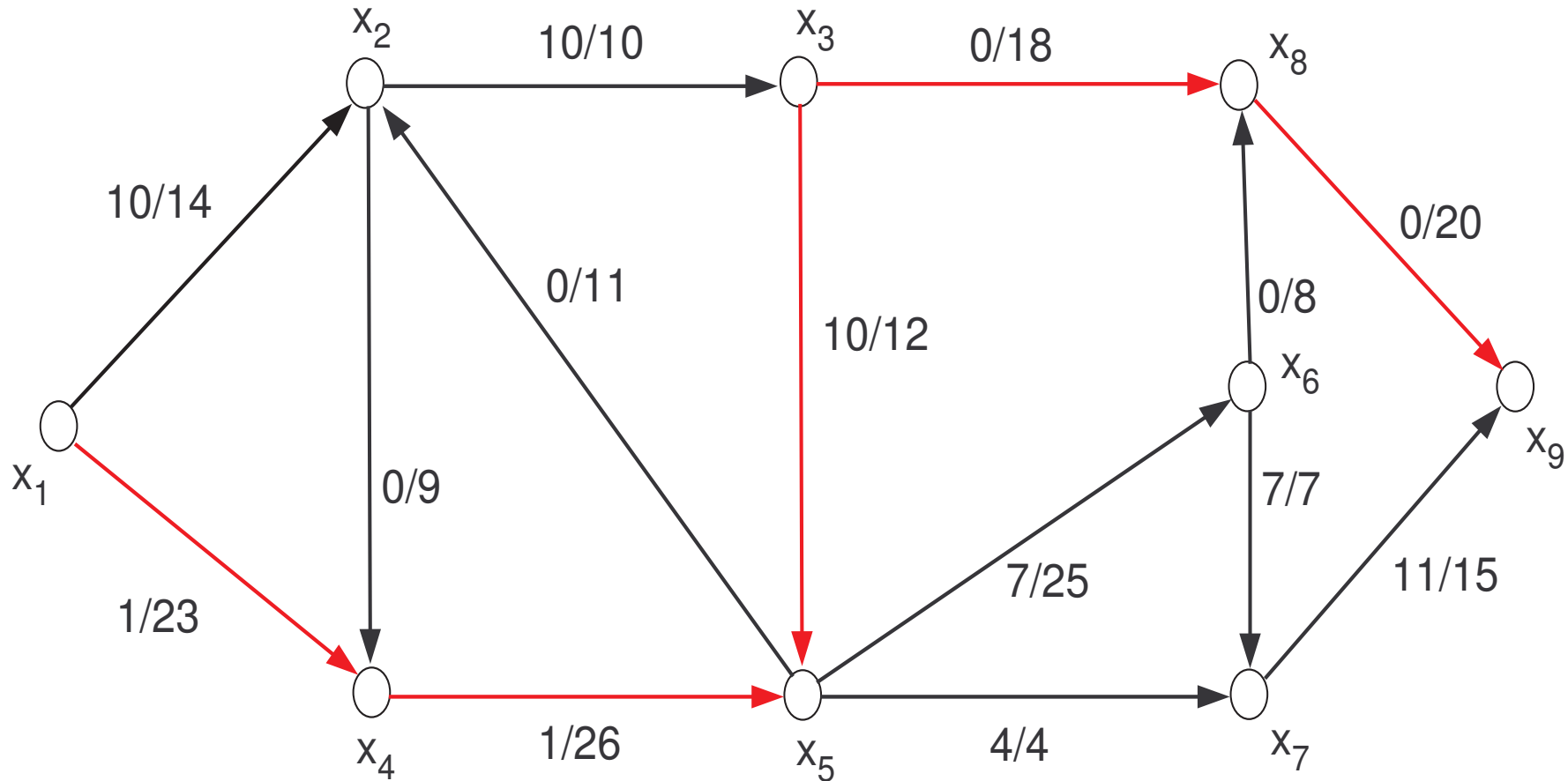
$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 4, & p(x_4) &:= +x_1, \delta(x_4) := 22, & p(x_5) &:= +x_4, \delta(x_5) := 22, \\
 p(x_6) &:= +x_5, \delta(x_6) := 18, & p(x_3) &:= -x_5, \delta(x_3) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10 \\
 p(x_9) &:= +x_8, \delta(x_9) := 10
 \end{aligned}$$

Ford-Fulkerson Algorithm



$$\begin{aligned}
 p(x_2) &:= +x_1, \delta(x_2) := 4, & p(x_4) &:= +x_1, \delta(x_4) := 22, & p(x_5) &:= +x_4, \delta(x_5) := 22, \\
 p(x_6) &:= +x_5, \delta(x_6) := 18, & p(x_3) &:= -x_5, \delta(x_3) := 10, & p(x_8) &:= +x_3, \delta(x_8) := 10 \\
 p(x_9) &:= +x_8, \delta(x_9) := 10
 \end{aligned}$$

Ford-Fulkerson Algorithm

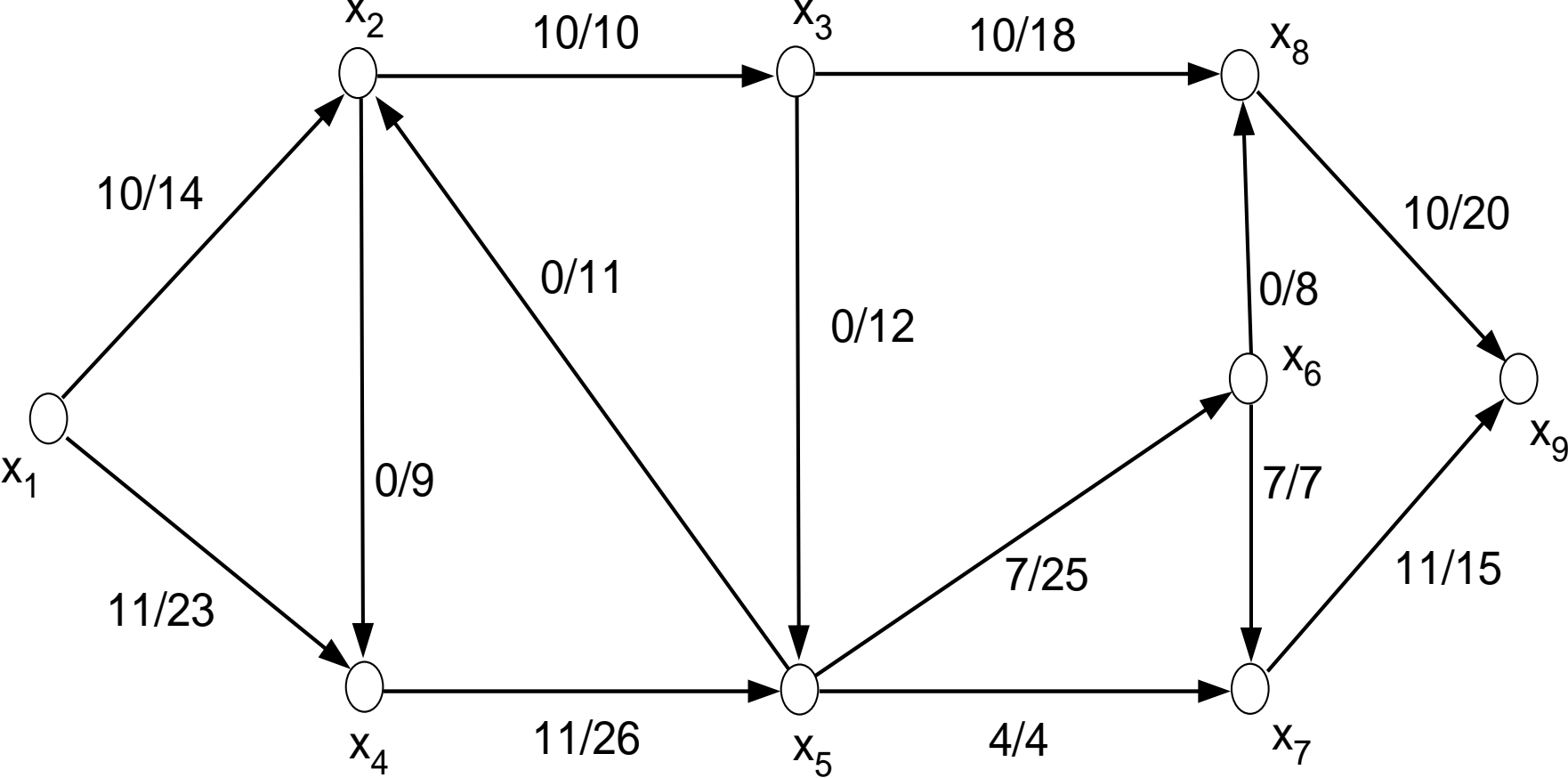


$$p(x_2) := +x_1, \delta(x_2) := 4, \quad p(x_4) := +x_1, \delta(x_4) := 22, \quad p(x_5) := +x_4, \delta(x_5) := 22,$$

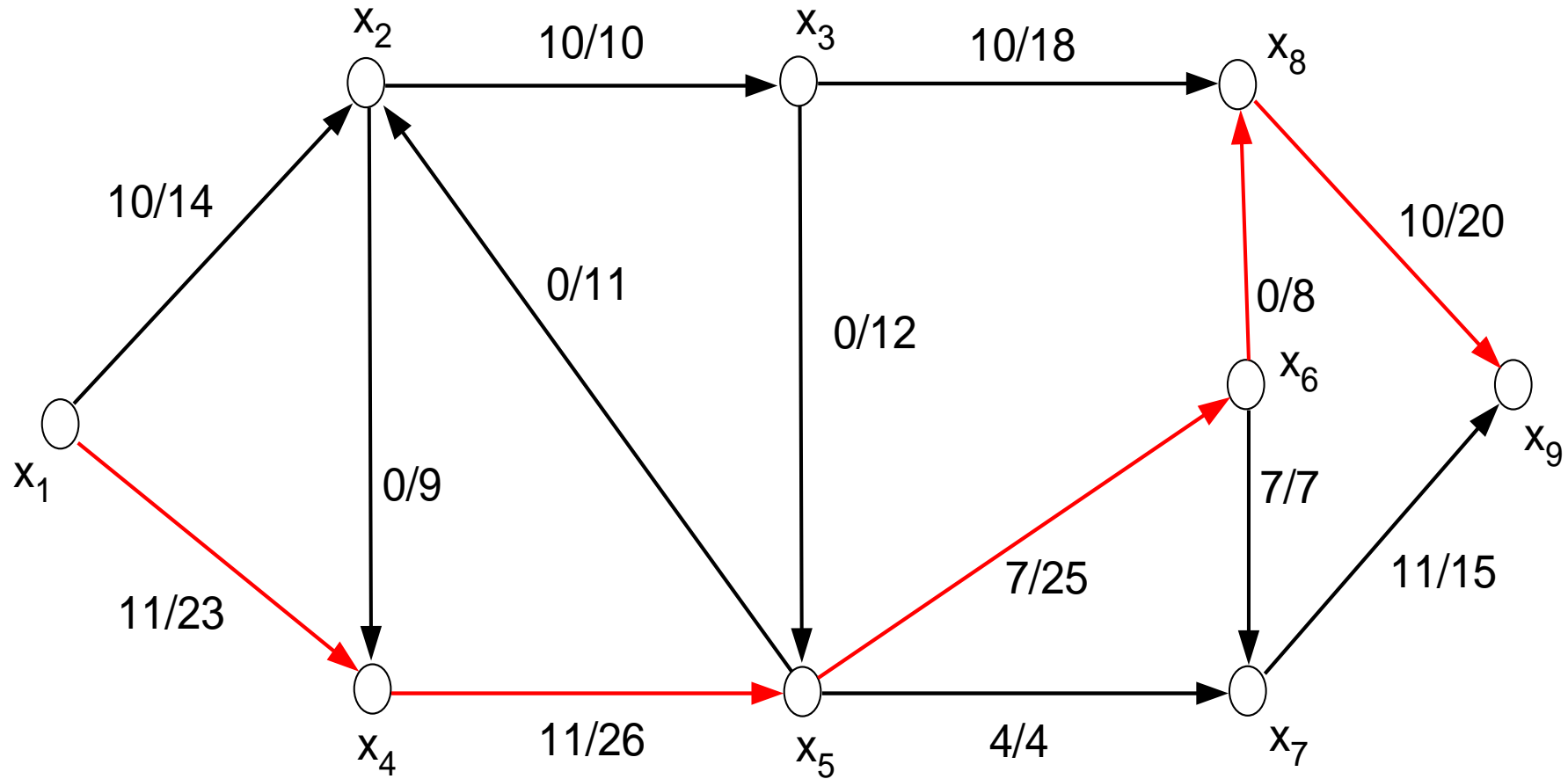
$$p(x_6) := +x_5, \delta(x_6) := 18, \quad p(x_3) := -x_5, \delta(x_3) := 10, \quad p(x_8) := +x_3, \delta(x_8) := 10$$

$$p(x_9) := +x_8, \delta(x_9) := 10, \quad x_1 \xrightarrow{+10} x_4 \xrightarrow{+10} x_5 \xleftarrow{-10} x_3 \xrightarrow{+10} x_8 \xrightarrow{+10} x_9$$

Ford-Fulkerson Algorithm



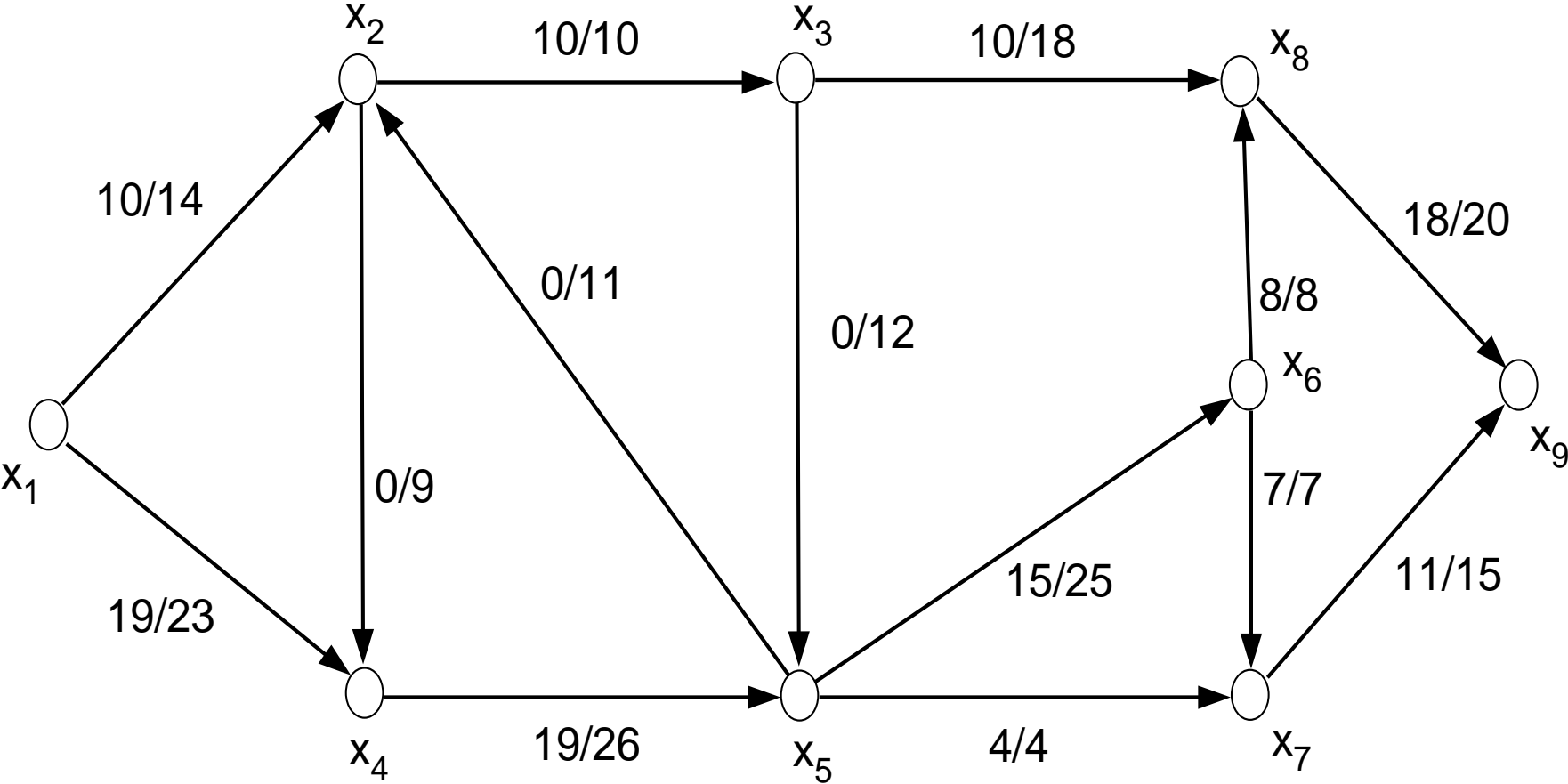
Ford-Fulkerson Algorithm



$x_1 \longrightarrow x_4 \longrightarrow x_5 \longrightarrow x_6 \longrightarrow x_8 \longrightarrow x_9$

$$\phi(e) = \delta(x_9) = 8$$

Ford-Fulkerson Algorithm



Ford-Fulkerson Algorithm

