Predicate Calculus pt. 4

Exercise 5 (maybe) and exercises 6-8 from the last sheet.

Exercise 1 *If the propositional formulas* $\phi_0(p_1, \ldots, p_n)$ *and* $\phi_1(p_1, \ldots, p_n)$ *are equivalent,*[1] *where* $p_1, \ldots p_n$ *are propositional variables, then so are* $\phi_0(f_1, \ldots, f_n)$ *and* $\phi_1(f_1, \ldots, f_n)$ *for arbitrary* L*-formulas* $f_1, \ldots, f_n$.

Set Theory

Exercise 2 *Show that using the standard definition of an ordered pair,*[2] *it is provable (in* ZFC*) that*

$$\forall x, y, x', y' \; (x, y) = (x', y') \rightarrow x = x' \wedge y = y'.$$

Exercise 3 *Define the ordered pair of* x *and* y *(alternatively) by*

$$(x, y) = \{x, \{y\}\}.$$

Does that work - in the sense of an analogous result to that of the previous exercise holding?

Exercise 4 *Define the ordered pair of* x *and* y *(alternatively) by*

$$(x, y) = \{\{\{x\}, \emptyset\}, \{\{y\}\}\}.$$

Does that work?

Exercise 5 *Define the ordered pair of* x *and* y *(alternatively) by*

$$(x, y) = \{x, \{x, y\}\}.$$

Does that work?

Exercise 6 *Try to find a suitable definition for the ordered triple* (x, y, z) *such that* (x, y, z) ZFC*-provably exists and such that*

$$(x, y, z) = (x', y', z') \rightarrow x = x', y = y', z = z'.$$

Exercise 7 *Show that if* ZFC *is consistent, then* ZFC *has a model* M *which is not well-founded.*[3]

Hint: Use the fact that arbitrarily long finite $\in$-sequences exist together with the compactness theorem.

[1] this means that they take the same truth value under all assignments of free variables
[2] $(x, y) = \{\{x\}, \{x, y\}\}$
[3] but of course M satisfies the axiom of foundation