Set Theory pt. 5

Exercises 4-10 from last time.

Exercise 1 *Show that the power set of ω has the same cardinality as the set of real numbers $\mathbb{R}$.*

Exercise 2 [1] *Let* DLO *(dense linear order without endpoints) be the theory in the language $\{<, =\}$ that consists of the following sentences:*

- *for all x and y, exactly one of the following three holds:*
  $x < y$, $y < x$ or $x = y$.
- *$\forall x, y, z\ x < y \wedge y < z \to x < z$ (transitivity).*
- *$\forall x\, \exists y\ y < x$, $\forall x\, \exists y\ y > x$ (no least and no largest element).*
- *$\forall x < y\, \exists z\ x < z < y$ (density).*

Show that every countable model $(M, <)$ of DLO *is isomorphic to $(\mathbb{Q}, <)$, the rationals with the standard order.*

Hint: Use a back-and-forth argument as in the construction of β^* in the proof of exercise 8 from the "Set Theory pt. 2" sheet.

Exercise 3 *Show that if A is a class of ordinal numbers such that*

$$\forall \alpha \exists \beta \in A\ \beta > \alpha,$$

then A is not a set.

[1] actually, this is not set, but model theory