Models of Set Theory II – Winter 2019/20

Peter Holy – Problem Sheet 7

Problem 1: Show that every lottery sum of proper forcing notions is proper.

Problem 2: Show that if κ is supercompact, then

1. κ is inaccessible, and
2. $\{\alpha < \kappa \mid \alpha \text{ is inaccessible}\}$ is a stationary subset of κ.

Note: In Problem 2, you should of course only use the definition of supercompactness that was introduced in the lecture.

Problem 3: Finish the proof of Lemma 9.3 by showing that, in the context of the proof of that lemma given in the lecture, $j^*\colon M[G] \to N[H]$, defined by letting, for $\tau \in M$,

$$j^*(\tau^G) = j(\tau)^H$$

is an elementary embedding such that $j^*(G) = H$ and $j^* \supseteq j$.

Problem 4: Verify the following:

0. If κ is a regular and uncountable cardinal, and T is a normal κ-tree with no antichains of size κ, then T has no κ-branch.
1. Every normal tree T is isomorphic to a tree $\bar{T}$ of sequences ordered by extension, closed under initial segments, and such that

$$\bar{T}_\beta = \{t \in \bar{T} \mid \mathrm{dom}(t) = \beta\}.$$

2. Show that if there is a κ-Suslin tree, and $\kappa = \lambda^+$ is a successor cardinal, then there is a normal κ-Suslin tree such that the following property (i)* holds:

   (i)* : Every node in T has λ-many immediate successors.