Transformable Luminaire Design

From digital sketch to fabrication through computation and simulation

Peter Ferschin, Georg Suter, Marco Palma, Ingrid Erb, David Hahn, Bálint Kovács, Georg Nawratil, Kiumars Sharifmoghaddam
TU Wien, Center for Geometry and Computational Design
{peter.ferschin|georg.suter|marco.palma}@tuwien.ac.at

Advanced computational design tools can help architectural and product designers to create novel and innovative designs. In this paper, we describe how advanced tools from research projects may be used together to design, simulate, and fabricate transformable luminaires. These tools support rapid design and simulation iterations to converge towards a realizable, usable and aesthetically design, which negotiates real-world constraints such as production costs, manufacturing time and material properties. We report on our experiences with integrated design and production workflows from teaching a digital design and production class, asking students to design and produce a luminaire based on a given production infrastructure. The design process starts with a conceptual part, where design intentions and basic ideas are explored with a 3D sketching tool. Students then develop parametric models by determining independent and dependent design parameters. As a required feature, the luminaire should have a transformable screen designed by a generator for flexible quad-surfaces. Real-time rendering tools allow for a fast, visual evaluation of these designs. After selecting the most suitable design regarding the design intention, students evaluate production feasibility and iteratively update their design until all production constraints are fulfilled. We describe the didactic and technical concepts and conclude with a discussion of open issues.

Keywords: *Digital Sketch, Light Simulation, Computational Fabrication, Parametric Design, Kinetic Structures, Architectural Education.*

INTRODUCTION

Advanced digital design and fabrication technologies provide significant opportunities for designers to realize innovative buildings and architectural products with improved qualitative and quantitative properties regarding shape, material, cost, usability, comfort, or sustainability. In this paper, we investigate the challenges of integrating design and fabrication technologies in a workflow that lets designers rapidly and seamlessly explore the feasibility of design alternatives with respect to heterogeneous constraints. The main focus of this work is the integration of results from the research project "Advanced Computational Design" (ACD) into an existing design–to-production workflow. Computational tools for 3D sketching, the generation of transformable structures and realistic light simulation were added into a workflow described by [Ferschin & Suter, 2020]. We adapt the linking of state-of-the-art visual scripting based parametric design tools and production planning simulation tools for laser cutting and 3d printing. We describe the necessary knowledge and skills that students in the Master of Architecture curriculum must acquire to productively use these tools with a luminaire design case study, see also [Marcos, et al., 2017]. Our work contributes to education in parametric design and digital fabrication by focusing on the skills- and knowledge- transfer needed for the integration of production planning simulation of multiple production methods in the early design process. Previous related work investigates design knowledge [Woodbury, 2010; Oxman, 2017] and pedagogical aspects of parametric design [Bacinoglu & Alacam, 2014], and the role of visual programming and scripting languages [Aish & Hanna, 2017; Celani & Vaz, 2012]. We reference further related work on digital workflows [Wortmann & Tunçer, 2017], design for manufacture and assembly [Austern, 2018], and multi-criteria design [Imbert, et al., 2012].

DESIGN AND PRODUCTION WORKFLOW

We created a workflow for students to develop and produce kinetic luminaires. Starting from different sketching strategies, students define parameters of their design and create parametric models. Light and intended effects are simulated, as well as the production process to ensure that a design meets production constraints. Design iterations by parameter adjustments explore lighting conditions and production feasibility. Figure 1 provides an overview of the design and production workflow, which we explain in detail in the following sections.

FROM SKETCH TO PARAMETRIC MODEL

The design idea for the luminaire should meet aesthetic requirements but also functional aspects, such as supported tasks or activities. Students need to consider the placement of the luminaire in a spatial situation as well as its type (i.e., hanging lamp, wall mount, floor or table lamp). We require also to

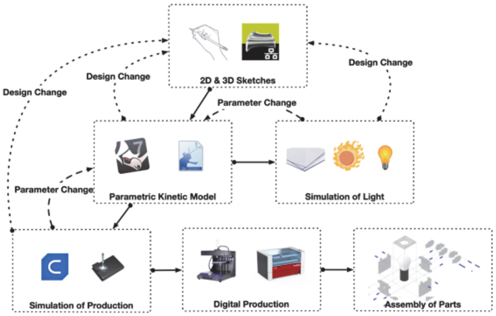

Figure 1: Overview of the design and production workflow

relate the luminaire screen and its transformation over time to a design inspiration that can be represented by reference images or sketches. To extend the basic conceptual ideas into the third dimension we provided a Mixed Reality sketching application (MR.Sketch) to develop design ideas directly in 3D space. A short introductory lecture on the use of MR.Sketch was given.

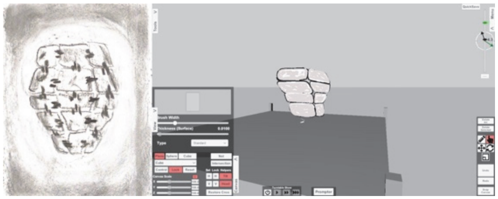

Figure 2: Sketching the “Armadillo Lamp” on paper 2D and in 3D with MR.Sketch

The 3D sketches (Figure 2) can be converted into several geometric data structures, like triangular and tetrahedral meshes or NURBS surfaces, which can be used to generate first drafts of parametric models. In our work we use the integration of the Scutes plugin (see following section) for Rhino/Grasshopper as part of the workflow to convert sketches to parametric models (Figure 3). The communication between MR.Sketch and Rhino is established by network connections. This allows the transfer of sketch lines as input for a parametric algorithm, as well as other additional parameters. As feedback to the sketching application, a mesh data structure is sent back to MR.Sketch, to visualize the parametric model. The exchange by streaming data allows a real-time interaction between our sketching environment and the parametric design tools.

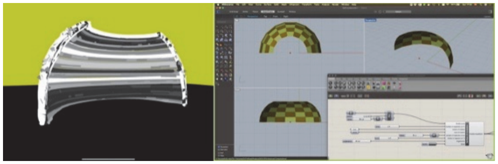

Figure 3: Linking MR.Sketch to the Grasshopper component Scutes

After the sketching phase students identified design parameters (independent parameters) that drive the main features of the design.

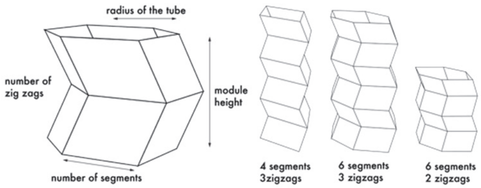

Figure 4: Design parameters and design variations of the "Ori Lamp"

In the example of the “Ori Lamp”, four independent parameters are identified for the T-hedral tube module (see Figure 4).

Since the design is generated procedurally with varying parameters, design variations can be explored easily and should be evaluated by the students to meet the intended design ideas, as well as the fabrication constraints.

DESIGN OF TRANSFORMABLE STRUCTURES

Transformable design has gained increased interest and developments in recent decades due to new applications in various fields, ranging from engineering and robotics to material sciences, since Chuck Hoberman's pioneering work in the 1990s [Hoberman, 2015]. Another creative discipline – beside kinetic art – that takes the aesthetic transformation of structures into account is architectural design, particularly for applications under functional aspects such as acoustics [Thün, et al., 2012] or shading/lighting [Barozzi, et al., 2016]. The latter also holds for our considered task of designing a transformable luminaire that exploits the potential offered by polyhedral lampshades composed of planar quads (PQ) hinged by rotary joints. PQ-meshes in the combinatorics of a square grid are generically rigid, but certain geometries allow for a 1-parametric change of the dihedral angles without any deformation of the PQ-panels. This so-called rigid-folding of the whole structure can be controlled by a single actuated rotary joint.

In addition to its functional aspect as an adaptive luminaire, the underlying PQ-mesh and its deformation should also meet aesthetic standards. Therefore, an intuitive access to its design space has to be provided. However, the rigid-foldability of PQ-meshes is not a property of the extrinsic geometry but of the intrinsic one, which is determined by the corner angles of the planar quads [Izmestiev, 2016]. Nonetheless, certain classes of rigid-foldable PQ-surfaces allow for direct access to their spatial shape through the use of control polylines. One class constitute so-called T-hedra [Sauer & Graf, 1931], whose geometry is completely encoded by three planar polylines (Figure 5). These are utilized within the design tool Scutes, a Rhino/Grasshopper plugin implemented in C#. Its geometric approach enables users to generate a T-hedron interactively and to visualize its rigid-foldability in real-time (by use of a slider) with a high accuracy. Other transformation simulator plugins (e.g., Kangaroo as used in [Agirbas, 2017]) have an accuracy/ speed trade-off due to their numerical nature, and even for an order of hundred points, they already show delays. Note that there are also further computational design tools available; e.g., [Tachi, 2010; Dang, et al., 2022; Montagne, et al., 2022], which are tailor-made for the generation of other special types of rigid-foldable PQ-meshes.

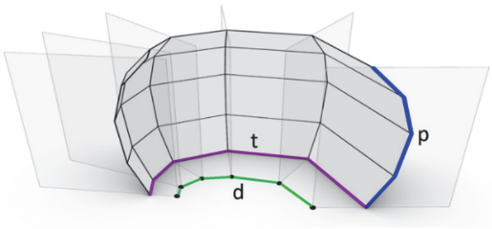

Figure 5: The polylines t and d are located in the base plane and the polyline p in a plane orthogonal to it in a way that p and t have the same start point.

The usability and design-related potential of Scutes were tested in the architectural seminar "Structure and Geometry" at the i.sd of the Uni-versity of Innsbruck in collaboration with Rupert Maleczek during the winter semester of 2021. The gained experiences resulted in the development of an add-on script to Scutes [Maleczek, et al., 2022], which facilitates the model-making process in the design phase by generating 3D-printable panels provided with snapping pin hinges for fast and, most importantly, accurate assembly. A modified version of this script was also offered to the students for the digital production of the transformable luminaires.

During a one-hour session, the students were introduced to T-hedra and Scutes. The session began with an explanation of the geometry of these PQ-surfaces and how they are related to the mentioned three polylines [Sharifmoghaddam, et al., 2021]. We then discussed well-known surface types (e.g., translational and moulding surfaces) that fall under the T-hedra class, as well as their flexion limits. Using illustrative examples, we demonstrated the functionality of Scutes and their flexion behaviour. In addition, we showcased digital and realized models having a variety of materials (e.g., paper, 3D printed PLA) and topologies; e.g., tubes [Sharifmoghaddam, et al., 2023]. For more details on Scutes, we refer to [Kovács, et al., 2022] and [Sharifmoghaddam, et al., 2021].

LIGHT SIMULATION OF THE LUMINAIRES

We provided students with a state-of-the-art real-time raytracing system (Tamashii) to generate interactive previews of the illumination produced by their luminaire design. Currently, data transfer between the parametric modelling tools and our rendering system uses a file-based interface.

In order to help students understand how this raytracing system works and what to expect from the resulting images, we gave a short introductory lecture covering the most important concepts. In particular, we first introduced the basics of path tracing, as well as a common material model for reflective and transmissive dielectric (i.e., non-conducting) materials. Finally, we discussed some trade-offs between computational cost, image noise, and accuracy.

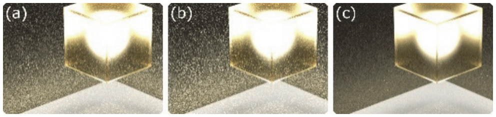

Figure 6: Rendering tests on a transparent open-bottom cube containing a light source; (a) recommended settings, (b) physically accurate but noisy settings, (c) exaggerated unphysical noise reduction.

The key advantage of a real-time raytracing approach is that students immediately see a preview of the resulting global illumination effects. While this preview is noisy at first, it can be progressively refined over time if desired. In contrast, most other real-time graphics engines only provide approximate lighting effects in a rasterization framework, whereas established raytracing software often does not allow interactive editing (i.e., changing the luminaire placement) while the rendering system is busy.

For this course, there are two components of the luminaire design of particular interest: the light source itself, and the optical properties of materials used to construct the luminaire. For the former, we provided students with an IES file [ANSI/IES, 2020] corresponding to a commercially available, reasonably priced, LED light bulb (the IES file is available from the manufacturer's website).

This file describes the radiant intensity of the bulb for all directions. Our system directly uses this data to produce an accurate rendering of the light source.

Most material models are derived from a statistical description of their micro-surface structure, primarily characterized by albedo colour and roughness (i.e., the variation of micro-surface normals relative to the large-scale surface normal). Additionally, the models prescribe which proportion of incident light is reflected, transmitted, or absorbed respectively. We use the well-known GGX material model [Walter, et al., 2007] throughout this course. During the introductory lecture, we explained this model, specifically the effects of transmission properties and surface roughness, to the students. They were then responsible for choosing appropriate values for all parameters of the model according to the materials they intended to use in their design.

Raytracing often introduces noise to the image due to Monte Carlo integration of the rendering equation [Pharr, et al., 2016]. A common approach is to "clamp" the maximal contribution for each light path, which alleviates noise produced by highly-specular materials. Doing so aggressively, however, causes unphysical energy loss and therefore a darker image. A subtler approach is to "filter glossy" reflections, i.e., to raise the material's roughness for indirect illumination paths. Intuitively, this method reduces the sharpness of reflections seen in objects, thereby also reducing noise along specular paths.
We first show the effects of the previously introduced noise-reduction methods on a simple test scene representative of our target application (Figure 6). Based on this test, we choose default render settings, which we suggested to the students; we also provided them with a living room scene [Bitterli, 2016].

The students had to insert their designed luminaire (including material parameters) into this scene in order to visually evaluate the resulting lighting effect. Note that our system allows the students to interactively move the luminaire around the scene and observe the visual effects in real time. Figure 7 shows a student-designed luminaire, placed in this scene, and rendered in our framework.

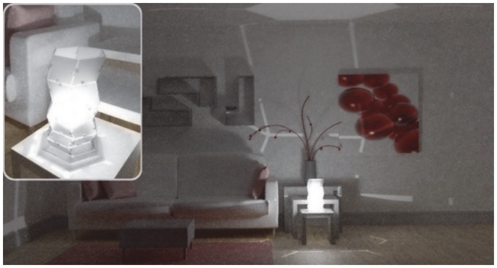

Figure 7: Rendered image of a luminaire, designed by students during this course, showing global illumination effects.

DIGITAL FABRICATION OF THE LUMINAIRES

Building on the experiences of graduate courses and post-graduate programs addressed to Digital Manufacturing [Stavrić, et al., 2012; Eversmann, 2017], we created a prototype- and production-based learning environment where students could develop design decisions not only by referring to set of abstract externalized constraints, but by physically engaging through observation and direct material feedback [Jenny, et al., 2022]. Given a tight time constraint and assuming students' little prior production experience, we defined a hybrid didactic approach that balances: 1) independent learning-by-doing; 2) focused technical knowledge transfer and 3) production-constrained design. As per 1, we empowered students with direct access to the production infrastructure as early as possible during the semester. After three training classes and one exam to evaluate safety-related issues and machine-specific hands-on management (laser cutters, 3d printers and CNC milling machines), students could gain knowledge of the fabrication laboratories to independently start to prototype. With respect to 2, we developed a series of vertical classes and related assignments conceived to let students familiarize with notions such as “machine tolerance” and “part-to-part fitting”. Regarding 3, we defined a set of production-related constraints to challenge the students’ creativity and to guide their decision-making process towards design trade-offs.

Designing with Tolerances

Students’ prior familiarity with CNC prototyping tools may not always match the design awareness which is necessary to 3d model two physical fitting parts, especially if produced with two different production methods. We therefore designed a series of assignments to address design-to-production core aspects, such as: understanding the dimensional discrepancy between a vector drawing and its material instantiation; reverse engineering a physical object by using precision measuring tools; or iteratively improving a design solution with the help of rapid prototyping. By addressing the final luminaire production task, the subject of such exercises addressed the design and prototyping of a rotational joint connecting a 2 mm thick acrylic panel and a 2 mm diameter acrylic rod. The students had to create a 3d model (with Rhinoceros and Grasshopper), post-process (with Ultimaker Cura and VisiCut), and prototype (3d print and laser cut) a joint solution to steadily connect the two parts. After a series of design development reviews, including several design and fabrication iterations, the initial designs evolved into properly engineered solutions meeting aesthetic, performance- and assembly-related requirements.

Figure 8 shows the “Armadillo” hinge, a double press-fit mechanism engineered to minimize the joint visibility from the luminaire frontside. In this solution, at each mesh non-naked edge, 3 hinges are connected to a 2 mm diameter acrylic rod (ensuring axial rotation), while each hinge connects to the corresponding panel by another acrylic rod (ensuring shear strain resistance).

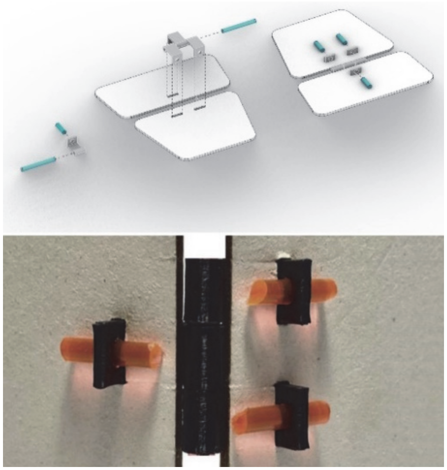

Figure 8: Design and fabrication of the “Armadillo” hinge.

Final design iteration (top) and 1:1: scale mock-up (bottom).

Production Goals and Constraints

The final deliverable of the course was a physical prototype of a transformable luminaire, produced by digital fabrication methods. We framed the luminaire as made of two mechanically connected bodies: a screen and a fixture. The screen, being conceived as the transformable element of the luminaire, is materialized as a set of panels connected to/by a set of rotational hinges. To ensure screen transformability, only one panel must connect to the fixture. Given the complexity of such a production task, we facilitated the luminaire production by means of a set of given geometry-, material- and process-related constraints. **Geometric Constraints**. The screen must be a transformable T-hedral mesh, whose PQ-faces are connected by 1DOF linear hinges. The hinges must be a simple vertical extrusion. The minimum amount of mesh faces must be 9, arranged in 3 rows and 3 columns. The geometric type of the fixture can be defined by the students according to the chosen luminaire typology. **Material Constraints.** The screen panels and the fixture must be fabricated with a single 2 mm thick PMMA sheet (max size

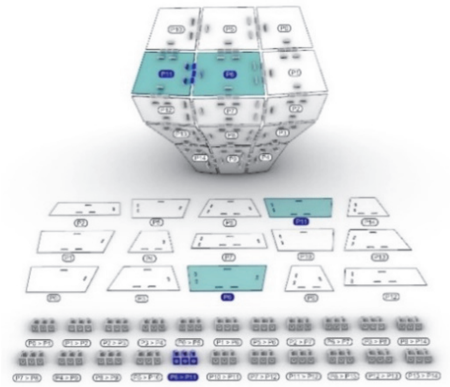

Figure 9: Output geometry of the computational tool for the design-to-production of the luminaire screen.

1270x720 mm). The use of off-the-shelf mechanical components is permitted. Gluing is not allowed, whereas joining through geometric fitting is strongly encouraged. **Process Constraints.** The production methods are limited to laser cutting (for the panels production) and FDM 3d printing (for the hinges production). The screen-fixture connection can be either laser-cut or 3d printed. The total cutting time is 45 minutes, while the total printing time is 360 minutes.

Design-to-Production Workflow

We supported the students with a computational tool for the parametric design and fabrication of the screen element. The tool is a custom-made Grasshopper plugin which expands the scope of prior work by [Maleczek, et al., 2022]. The tool relies on the assumption that the screen is formed by two fabrication-wise distinct parts, rather than by a unified panel-hinge solution. Furthermore, the tool was redesigned to seamlessly accommodate a variety of hinge design inputs.

The plugin receives two main geometric inputs (a T-hedral mesh representing the transformable panels array and a parametric planar curve representing the rotational hinge profile). Furthermore, a set of numerical values encoding design, fabrication and assembly tolerances (i.e., number of hinges per edge). A series of parametric functions map the input curve profile to the input 3d mesh and performs a sequence of geometric operations to generate a production-ready 3d model. The 3d model can be visualized in an assembled 3d configuration for design evaluation and in an unfolded 2d planar configuration ready to be exported to the post-processing software (Figure 9). To facilitate the assembly process, the plugin consistently labels each hinge triplet referring to the corresponding panel numbering and allows for an interactive digital simulation of the construction process. The tool provides a rough estimate of both cutting and printing times in the Grasshopper environment. More accurate production times are later evaluated in Ultimaker Cura and in VisiCut, which also support for machine code post-processing [Ferschin & Suter, 2020].

RESULTS

We selected three luminaire designs (Figure 10) to demonstrate the flexibility and robustness of our design-to-production workflow. Our tools allowed the students to explore a variety of design ideas, materials, lighting scenarios as well as kinematic solutions.

The "Armadillo" lamp, a 5x3 T-hedral mesh, is a wall-mounted lamp acting as a mirror in the daytime and as semi-transparent filter when backlit. Additionally, its activation mechanism allows the screen to tilt relative to the vertical wall, increasing the range of lighting possibilities.

The "Hexlight" lamp is assembled from two identical 3x3 T-hedra, where corresponding outer edges of the central strips are hinged to a loop structure, which is still rigid-foldable. It is a table lamp that can be adjusted to either fully expose or fully hide the light source. The "Ori" lamp is a floor lamp inspired by the Japanese art of paper folding and is designed to provide a diffuse atmospheric light. It consists of a

T-hedral tube with six ring-segments where a sliding hexagonal profile base curve connects to the foldable screen allowing its form to be compressed by pushing from the top as well as to be expanded by pressing from the sides.

CONCLUSIONS

The variety of luminaires that were designed and produced by students in 12 weeks suggests that our goal to teach parametric design and fabrication in combination with recent research results - to expand the existing workflow with 3D sketching (MR.Sketch), to include transformable structures (Scutes) and to provide a light simulation tool (Tamashii) for additional design evaluation - was fulfilled. It turned out, that the product design task of designing and producing luminaires, lead to a deeper understanding in a complete workflow from digital design to production. The main insights from the students were how bad design decisions influence the quality of the luminaire, such as instable constructions or errors from the produced parts, that needed to be corrected at the assembly process. The project helped also to understand the value of a well-designed parametric model, so adjustments to meet the design goal or the production constraints could be made by parameter changes. In addition, it was possible to learn that design at different scales is subject to the same design considerations and that the experience gained in product design could well be transferred to the scale of larger objects.

In the future, we plan the following improvements in teaching: We will start earlier with the design task, reorganize the previous lectures and their exercises to better support the design task. We plan to partially switch the teaching format to a set of design workshops, which would allow better feedback than the current approach based on weekly lectures. The introduction of research tools should be accompanied with additional teaching material, that will also help in the evaluation of new research. We plan also to establish a direct integration of Scutes into MR.Sketch. To reduce the workload for the students, we will prepare more assets for the design task (e.g., accurate lamp libraries, scanned material libraries). We plan to provide tools for scanning optical parameters of materials used in the production process. Finally, we intend to utilize the acquired knowledge for the development of more

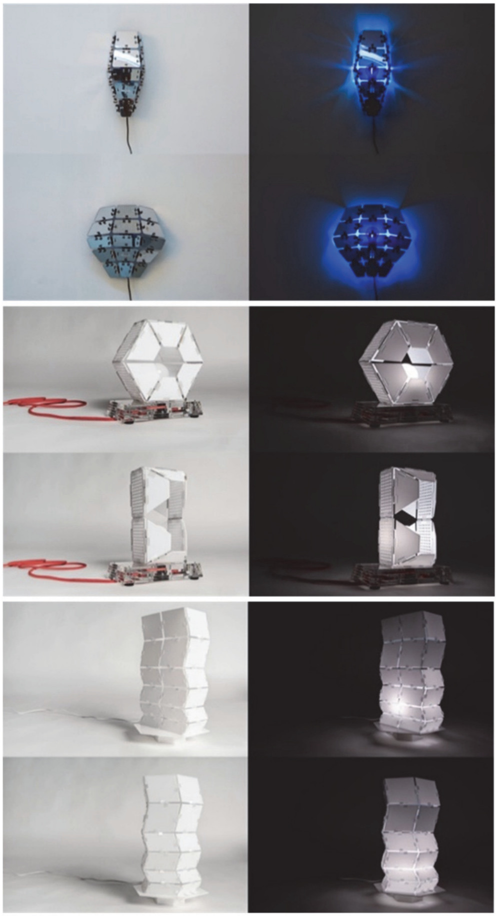

Figure 10: Photo–graphs of the fabricated luminaires in two extreme kinematic configurations as well as in their lit and unlit condition. From top to bottom, the “Armadillo” wall lamp, the “Hexlight” table lamp and the “Ori” floor lamp.

complex architectural, product, or interior design exercises.

ACKNOWLEDGEMENTS

This work received funding from the Austrian Science Fund (FWF) project F 77 (SFB “Advanced Computational Design”). Thanks to Rupert Maleczek for providing us the add-on script to “Scutes”. Students from “Digital Design and Production”: Moritz Batz, Julian Fellner, Tim Guckelberger (Amardillo Lamp), Natalia Anisimova, Benjamin Kislich, Martin Stockreiter (Hexlight), Gjulabije Dauti, Elena Ruppitsch, Fatima Sena Yargi (Ori Lamp).

REFERENCES

Agirbas, A. (2017). The Use of Simulation for Creating Folding Structures - A Teaching Model. *Proc. of the 35th eCAADe*, 325–332. doi:10.52842/conf.ecaade.2017.1.325

Aish, R., & Hanna, S. (2017). Comparative evaluation of parametric design systems for teaching design computation. *Design Studies, 52*, 144-172.

ANSI/IES. (2020). IES Standard File Format for Photometric Data, ANSI/IES LM-63-19. *webstore.ansi.org/Standards/IESNA/ANSIIESLM6319 (2023-03-24)*.

Austern, G. C. (2018). Rationalization methods in computer aided fabrication: A critical review. *Automation in Construction, 90*, 281-293.

Bacinoglu, Z., & Alacam, S. (2014). A Context Based Approach to Digital Architectural Modelling Education. *Proceedings of CAADRIA*. doi:10.52842/conf.caadria.2014.811

Barozzi, M., Lienhard, J., Zanelli, A., & Monticelli, C. (2016). The Sustainability of Adaptive Envelopes: Developments of Kinetic Architecture. *Procedia Engineering, 155*, 275-284. doi:10.1016/j.proeng.2016.08.029

Bitterli, B. (2016). Rendering resources. *https://benedikt-bitterli.me/resources (2023-03-24)*.

Celani, G., & Vaz, C. E. (2012). CAD Scripting and Visual Programming Languages for Implementing Computational Design Concepts: A Comparison from a Pedagogical Point of View. *Int. J. Architectural Computing, 10*, 121-137.

Dang, X., Feng, F., Plucinsky, P., James, R. D., Duan, H., & Wang, J. (2022). Inverse design of deployable origami structures that approximate a general surface. *International Journal of Solids and Structures, 234-235, 111224*. doi:10.1016/j.ijsolstr.2021.111224

Eversmann, P. (2017). Digital Fabrication in Education-Strategies and Concepts for Large-Scale Projects. *Proc. of the 35th eCAADe Conference*, 333-342.

Ferschin, P., & Suter, G. (2020). Integrated Parametric Design and Production Planning: A Luminaire Design Case Study. *Proc. Symposium on Simulation for Architecture and Urban Design*, 565-572.

Hoberman, C. (2015). Transformable: Building Structures that change themselves. *Building Dynamics.* Routledge. doi:10.4324/9781315763279-13

Imbert, F., Frost, K. S., Fisher, A., Witt, A., Tourre, V., & Koren, B. (2012). Concurrent Geometric, Structural and Environmental Design: Louvre Abu Dhabi. *Advances in Architectural Geometry*, 77-90.

Izmestiev, I. (2016). Classification of Flexible Kokotsakis Polyhedra with Quadrangular Base. *International Mathematics Research Notices, 2017*, 715-808. doi:10.1093/imrn/rnw055

Jenny, D., Mayer, H., Aejmelaeus-Lindström, P., Gramazio, F., & Kohler, M. (2022). A Pedagogy of Digital Materiality: Integrated Design and Robotic Fabrication Projects of the Master of Advanced Studies in Architecture and Digital Fabrication. *Architecture, Structures and Construction, 2*, 649–660.

Kovács, B. I., Sharifmoghaddam, K., Jauk, J., Erb, I., Stavric, M., Nawratil, G., & Ferschin, P. (2022).

Integrative Mixed Reality Sketching. *Proceedings of the Creative Construction e-Conference 2022*, (pp. 233–240). doi:10.3311/CCC2022-030
Maleczek, R., Sharifmoghaddam, K., & Nawratil, G. (2022). Rapid prototyping for non-developable discrete and semi-discrete surfaces with an overconstrained mobility. *Proceedings of the IASS Annual Symposium 2022 and the 13th Asian-Pacific Conference on Shell and Spatial Structures*, (pp. 2302–2313).
Marcos, C. L., Capone, M., & Lanzara, E. (2017). Digitally Conscious Design - From the Ideation of a Lamp to its Fabrication as a Case Study. *Proc. 35th International Conference on Education and Research in Computer Aided Architectural Design in Europe (eCAADe).* doi:10.52842/conf.ecaade.2017.2.219
Montagne, N., Douthe, C., Tellier, X., Fivet, C., & Baverel, O. (2022). Discrete Voss surfaces: Designing geodesic gridshells with planar cladding panels. *Automation in Construction, 140*, 104-200. doi:10.1016/j.autcon.2022.104200
Oxman, R. (2017). Thinking difference: Theories and models of parametric design thinking. *Design Studies, 6, 52*.
Pharr, M., Jakob, W., & Humphreys, G. (2016, October). *Physically Based Rendering: From Theory to Implementation (3rd ed.)*. Morgan Kaufmann Publishers Inc.
Sauer, R., & Graf, H. (1931). Über Flächenverbiegung in Analogie zur Verknickung offener Facettenflache. *Mathematische Annalen, 105*, 499–535.
Sharifmoghaddam, K., Maleczek, R., & Nawratil, G. (2023). Generalizing rigid-foldable tubular structures of T-hedral type. Retrieved from arXiv:2301.09969: https://arxiv.org/abs/2301.09969
Sharifmoghaddam, K., Nawratil, G., Rasoulzadeh, A., & Tervooren, J. (2021). Using flexible trapezoidal quad-surfaces for transformable design. *Proc. IASS Annual Symposium 2020/21 and the 7th International Conference on Spatial Structures, 21*, 1–13.
Stavrić, M., Stokić, D., & Ilić, M. (2012). Architectural Scale Model In Digital Age–Design Process, Representation And Manufacturing. *ECAADE 2012*, 33-42.
Tachi, T. (2010). Freeform Rigid-Foldable Structure Using Bidirectionally Flat-Foldable Planar Quadrilateral Mesh. *Advances in Architectural Geometry*, 87-102. doi:10.1515/9783990433713-007
Thün, G., Velikov, K., Ripley, C., Sauvé, L., & McGee, W. (2012). Soundspheres: Resonant Chamber. *ACM SIGGRAPH 2012 Art Gallery*, 348-357. doi:10.1145/2341931.2341936
Walter, B., Marschner, S. R., Li, H., & Torrance, K. E. (2007). Microfacet Models for Refraction through Rough Surfaces. *Proc. 18th Eurographics Conference on Rendering Techniques*, 195–206.
Woodbury, R. (2010). *Elements of Parametric Design.* Routledge.
Wortmann, T., & Tunçer, B. (2017). Differentiating parametric design: Digital workflows in contemporary architecture and construction. *Design Studies, 52*, 173-197.