

# CLONES ON HOMOGENEOUS STRUCTURES AND SCHAEFER'S THEOREM FOR GRAPHS

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My talk will be an opera in a prologue, three acts, and an epilogue.

**Prologue.** In a café in Paris, young mathematician Manuel Bodirsky tells his friend Michael Pinsker that he is being haunted by the *graph satisfiability problem*, a generalization of the Boolean satisfiability problem. For a fixed set  $\Psi$  of allowed formulas in the language of graphs, an instance of the computational problem  $\text{Graph-SAT}(\Psi)$  consists of a set  $W$  of vertices and statements  $\psi_1, \dots, \psi_n$  about these vertices, where each statement is a formula from  $\Psi$ . The task is to decide whether there exists a graph on  $W$  such that all statements become true. The friends discuss how to translate the problem into a constraint satisfaction problem for a template with a definition in the countably infinite random graph, and conjecture that the problem is either NP-complete or in P, for all  $\Psi$ . They set out to study structures with a first-order definition in (=“reducts of”) the random graph.

**Act I.** In a small village in Britain, combinatorialist Peter Cameron is happy because he just proved that the dense linear order has, up to first-order equivalence, 5 reducts. Model-theorist Simon Thomas enters the stage, and sings an air in which he states that the same is true for the random graph. The other villagers count the number of reducts of some other structures, and the act ends with Simon Thomas claiming that the number of reducts of any homogeneous structure in a finite language is finite.

**Act II.** Manuel and Michael have received a magic Ramsey-theoretic potion produced by Jaroslav Nešetřil and Vojtech Rödl that allows them to find regular patterns in the behavior of functions on reducts of structures with the *Ramsey property*, in particular the random graph. They hear about Thomas' conjecture and want to apply their elixir to arbitrary homogeneous structures with the Ramsey property, but a technical obstacle prevents their success.

**Act III.** The two friends have decided to consult the three wise men Alexander Kechris, Vladimir Pestov, and Stevo Todorćević to translate the obstacle into a strange language called topological dynamics. Being approached in this language, young tenor Todor Tsankov does not hesitate one second to resolve the problem. Relieved, the protagonists sing some theorems about minimal reducts of homogeneous structures with the Ramsey property, and dream of proving Thomas' conjecture for such structures in the future.

**Epilogue.** Back in Paris, and with the new tools at hand, Manuel and Michael prove the complexity dichotomy for  $\text{Graph-SAT}(\Psi)$ .

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