Reducts of Ramsey structures: the canonical approach

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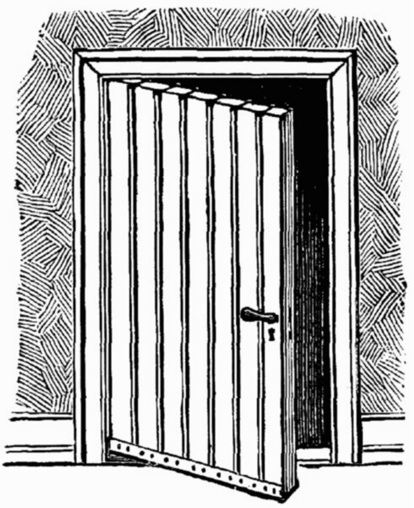
Outline



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Problem

Classify the reducts of Δ.

We call Δ the *base structure*.

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We factor this quasiorder
by the equivalence relation of fo-interdefinability,
and obtain a complete lattice.

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Observe:

Primitive positive (pp) interdefinability is finer than
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existential (ex) interdefinability is finer than
first order (fo) interdefinability.

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In fact:

The lattice corresponding to fo-definability is a factor of
the lattice corresponding to ex-definability is a factor of
the lattice corresponding to ep-definability is a factor of
the lattice corresponding to pp-definability.

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This talk: Method for pp (and ep - submethod).

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STOP!

In practice helps also for fo.

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For our method, we will need even “more” than homogeneity in a finite language:

The Ramsey property

Example: The dense linear order

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Denote by $(\mathbb{Q}; <)$ be the order of the rationals, and set

$$\begin{aligned}
\text{betw}(x,y,z) :=& \{(x,y,z) \in \mathbb{Q}^3 : x < y < z \text{ or } z < y < x\} \\
\text{cycl}(x,y,z) :=& \{(x,y,z) \in \mathbb{Q}^3 : x < y < z \text{ or } z < x < y \\
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5. Γ is first-order interdefinable with $(\mathbb{Q};=)$.

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5. Γ is first-order interdefinable with $(V; =)$.

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The homogeneous universal k-graph has $2^k + 1$ reducts up to fo-interdefinability.

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Theorem (Junker, Ziegler '08)

$(\mathbb{Q}; <, 0)$ has 116 reducts up to fo-interdefinability.

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Depressing fact (Horváth, Pongrácz, P. '11)

The random graph with a constant has too many reducts up to fo-interdefinability.

Thomas' conjecture

Conjecture (Thomas '91)

Let Δ be homogeneous in a finite language.

Then Δ has finitely many reducts up to fo-interdefinability.

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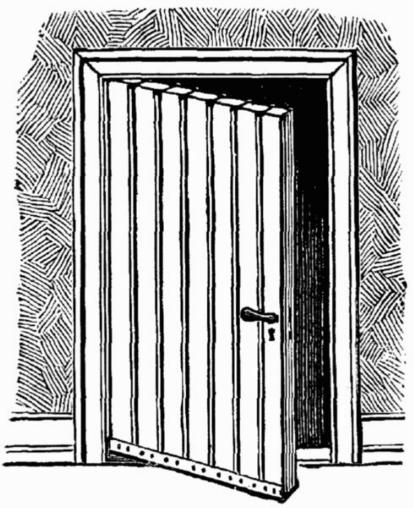

Functions on homogeneous structures

Permutation groups

Permutation groups

Theorem (Ryll-Nardzewski)

Let Δ be ω-categorical.

The mapping

$$\Gamma \mapsto \mathrm{Aut}(\Gamma)$$

is a one-to-one correspondence between
the *first-order closed* reducts of Δ and
the *closed permutation groups* containing $\mathrm{Aut}(\Delta)$.

first order closed = contains all fo-definable relations

Monoids

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Theorem (follows from the Homomorphism preservation thm)

Let Δ be ω-categorical.

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$$\Gamma \mapsto \mathrm{End}(\Gamma)$$

is a one-to-one correspondence between
the *existential positive closed* reducts of Δ and
the *closed transformation monoids* containing $\mathrm{Aut}(\Delta)$.

A monoid of functions from Δ to Δ is *closed* iff
it is closed in the Baire space Δ^Δ.

Clones

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Theorem (Bodirsky, Nešetřil '03)

Let Δ be ω-categorical. Then

$$\Gamma \mapsto \mathrm{Pol}(\Gamma)$$

is a one-to-one correspondence between
the *primitive positive closed* reducts of Δ and
the *closed clones* containing $\mathrm{Aut}(\Delta)$.

A clone is a set of finitary operations on Δ which

- contains all projections $\pi_i^n(x_1, \ldots, x_n) = x_i$, and
- is closed under composition.

$\mathrm{Pol}(\Gamma)$ is the clone of all homomorphisms from finite powers of Γ to Γ.

A clone C is closed if for each $n \geq 1$, the set of n-ary operations in C is a closed subset of the Baire space Δ^{Δ^n}.

Groups, Monoids, Clones

For ω-categorical Δ:

Reducts up to **fo-interdefinability** $\leftrightarrow$
closed **permutation groups** $\supseteq \text{Aut}(\Delta)$;

Reducts up to **ep-interdefinability** $\leftrightarrow$
closed **monoids** $\supseteq \text{Aut}(\Delta)$

Reducts up to **pp-interdefinability** $\leftrightarrow$
closed **clones** $\supseteq \text{Aut}(\Delta)$.

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Let $G := (V; E)$ be the random graph.

Let $\bar{G}$ be the graph that arises by switching edges and non-edges.

Let $- : V \to V$ be an isomorphism between G and $\bar{G}$.

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Theorem (Thomas '91)

The closed groups containing $\mathrm{Aut}(G)$ are the following:

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5. The full symmetric group S_V.

How to classify all reducts up to . . .-interdefinability?

Climb up the lattice!

Canonical functions between structures

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Let Δ, Λ be structures.

Definition

$f : \Delta \to \Lambda$ is *canonical* iff
for all tuples $(x_1, \ldots, x_n), (y_1, \ldots, y_n)$ of the same type in Δ
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Possible types: edge, non-edge, point.

Examples of canonical functions

General examples.

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Canonical functions induce functions on types.

If the structures Δ, Λ are homogeneous in a finite language, then there are just finitely many canonical behaviors for $f : \Delta \to \Lambda$.

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Definition

A class $\mathcal{C}$ of τ-structures is called a *Ramsey class* iff
for all $H, P \in \mathcal{C}$ there exists S in $\mathcal{C}$ such that $S \to (H)^P$.

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then all finite substructures of Δ have a copy in Δ on which f is canonical.

Thus: If Δ, Λ are homogeneous, then the closure of $\text{Aut}(\Lambda) \circ f \circ \text{Aut}(\Delta)$ in Λ^Δ contains a canonical function.

Finding canonical behaviour on G

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So every $f : G \to G$ *generates* (with $\mathrm{Aut}(G)$) a canonical function.

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We would like to fix $c_1, \ldots, c_n$ witnessing $f \notin \mathrm{Aut}(\Delta)$,
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Why don't you just do it?

Adding constants to Ramsey structures

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If Δ is Ramsey, is $(\Delta, c_1, \ldots, c_n)$ still Ramsey?

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Corollary

If Δ is ordered, homogeneous, and Ramsey, then so is $(\Delta, c_1, \ldots, c_n)$.

Canonizing functions on Ramsey structures

Proposition

If Δ is ordered Ramsey homogeneous finite language, $f : \Delta^k \to \Delta$, and $c_1, \ldots, c_n \in \Delta$, then f generates a function which

- is canonical as a function from $(\Delta, c_1, \ldots, c_n)^k$ to Δ
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Any element of the fixed point is canonical. □

The minimal monoids on the random graph

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Theorem (Thomas '96)

Let $f : G \to G$ a function
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(that is, it violates at least one edge or a non-edge.)

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Generalized to *minimal closed clones* (14) by Bodirsky, P. 2010.

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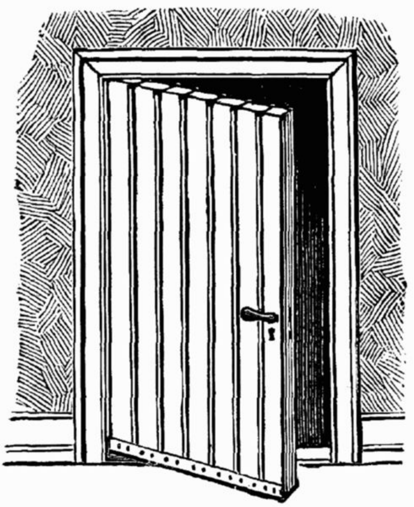
Non-trivial: arity bound!



Reducts of the random graph

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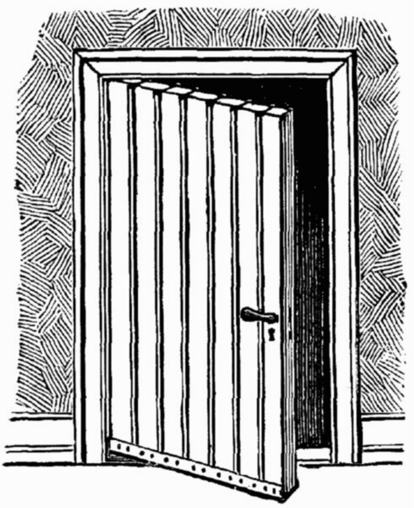
Interesting: works without knowing the relational descriptions.



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Theorem (Bodirsky, P., Tsankov '10)

Let Δ be

- ordered
- homogeneous
- Ramsey
- with finite language
- *finitely bounded*.

Then the following problem is decidable:

INPUT: Two finite language reducts Γ, Γ' of Δ.
QUESTION: Are Γ, Γ' pp (ep-) interdefinable?

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- Is fo-interdefinability decidable?

References

Reducts of Ramsey structures

by Manuel Bodirsky and Michael Pinsker

Reducts of the random partial order

by Péter P. Pach, Michael Pinsker, András Pongrácz, Gabriella Pluhár, Csaba Szabó