Algebraic and model-theoretic methods in constraint satisfaction

2nd session

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Technische Universität Wien / Université Diderot - Paris 7

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Doc-Course, Charles University Prague

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Outline reminder

Part I: CSPs / dividing the world /
pp definitions, polymorphism clones, ω-categoricity

Part II: pp interpretations / topological clones

Part III: Canonical functions, Ramsey structures / Graph-SAT

Part IV: Model-complete cores / The infinite tractability conjecture

Summary of last session

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For ω-categorical Γ: if Pol(Γ) $\subseteq$ Pol(Γ'),
then CSP(Γ') is polynomial-time reducible to CSP(Γ).

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Theorem (Bodirsky + Nešetřil ’03)

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A relation is pp definable over Γ iff
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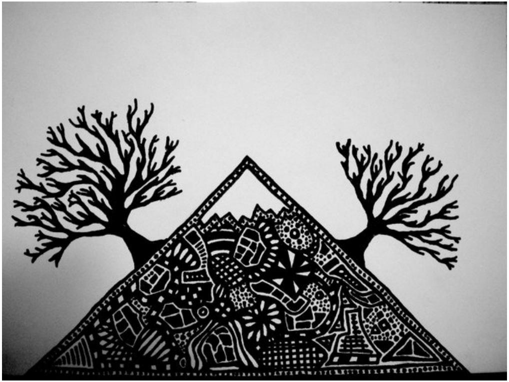
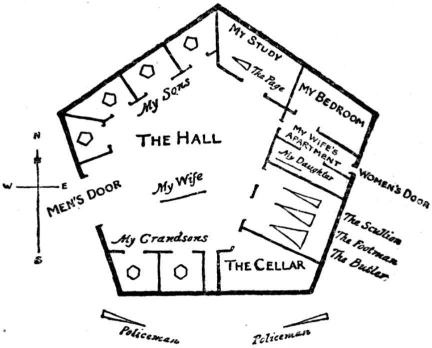


Part II:

pp interpretations / topological clones



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- finite Powers.

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- $\mathrm{H}(\mathbf{C})\ldots$ function clones obtained by factoring by congruence.

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Let Γ be ω-categorical. If Pol(Δ) contains a function clone which can be obtained from Pol(Γ) by H, S, P$^{\text{fin}}$, then

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Let Γ be ω-categorical. If $\mathrm{Pol}(\Delta)$ contains a function clone which can be obtained from $\mathrm{Pol}(\Gamma)$ by H, S, $\mathrm{P}^{\mathrm{fin}}$, then

- Δ is ω-categorical and $\mathrm{Pol}(\Delta)$ contains a clone in $\mathrm{HSP}^{\mathrm{fin}}(\mathrm{Pol}(\Gamma))$;
- $\mathrm{CSP}(\Delta)$ is polynomial-time reducible to $\mathrm{CSP}(\Gamma)$.

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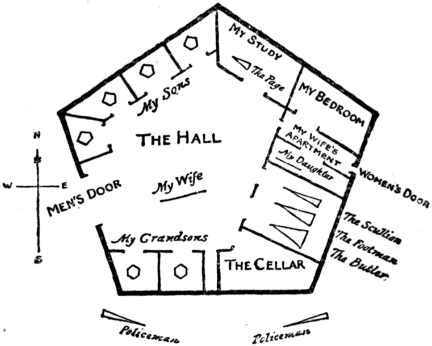
Proof sketch

- Subuniverses, congruence relations are pp-definable;
- Δ can be simulated ("pp interpreted") on pp-definable factor of pp-definable subset of finite power of Γ.



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Set of all finitary functions $\bigcup_n D^{D^n} \ldots$ sum space.

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For finite function clones: topology discrete.

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Let Δ, Γ be ω-categorical or finite. TFAE:

- Δ has a pp interpretation in Γ;
- there exists a continuous homomorphism $\xi\colon \mathrm{Pol}(\Gamma) \to \mathrm{Pol}(\Delta)$ whose image is dense in an oligomorphic function clone.

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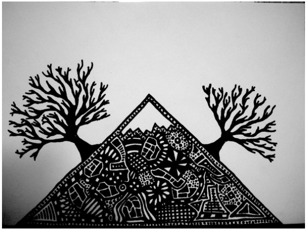
- there exists a continuous clone homomorphism $\xi\colon$ Pol(Γ) $\to$ Pol(Γ′) for any finite Γ′;
- all finite Γ′ have a pp interpretation in Γ.

“I am indeed, in a certain sense a Circle," replied the Voice,
“and a more perfect Circle than any in Flatland;
but to speak more accurately,
I am many Circles in one."



Part III: November 6th