The Equivalence of Two Dichotomy Conjectures for Infinite Domain CSPs

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ω-categorical: countable and $\Gamma^n/\mathrm{Aut}(\Gamma)$ is finite for all $n \geq 1$.

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- When is Pol(Γ) "rich" / "poor"?

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Henceforth assume ω-categoricity.

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Pol(Γ) $\not\rightarrow$ **P**, even after some preprocessing $\Leftrightarrow$ Pol(Γ) contains u, v, s:
$u(s(x, y, x, z, y, z)) = v(s(y, x, z, x, z, y))$ "Pseudo-Siggers".

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Dichotomy Conjecture (Bodirsky + P '11)

For a certain class of Γ, richness of Pol(Γ) forces CSP(Γ) into P.

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$\mathrm{Pol}(\Gamma) \dashrightarrow \mathbf{P}$ if there exists a mapping preserving linear identities (no nesting), uniformly continuous.

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If $\mathrm{Pol}(\Gamma) \dashrightarrow \mathbf{P}$, then CSP($\Gamma$) is NP-hard.

New Dichotomy Conjecture (Barto + Opršal + P '15)

For a certain class of Γ, new richness of $\mathrm{Pol}(\Gamma)$ forces CSP(Γ) into P.

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Any such Γ must have at least double exponential orbit growth:
For every $n \geq 1$, $\Gamma^n/\mathrm{Aut}(\Gamma)$ has at least 2^{2^n} elements asymptotically.

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Let Γ be first-order definable in a finitely bounded homogeneous structure. Then the following are equivalent:

- $\mathrm{Pol}(\Gamma) \not\to \mathbf{P}$ after preprocessing.
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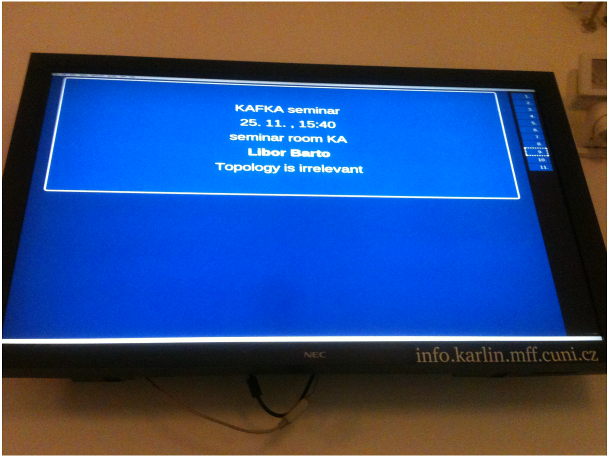
Examples:

- Temp-SAT problems (rational order)
- Graph-SAT problems (random graph)
- Poset-SAT problems (random partial order)


KAFKA seminar
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Thank you!