UNIQUENESS OF POLISH TOPOLOGIES ON ENDOMORPHISM MONOIDS OF COUNTABLE STRUCTURES

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PANGLOBAL ALGEBRA & LOGIC SEMINAR

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PART I : MOTIVATION & RESULTS

- ALGEBRAIC - TOPOLOGICAL STRUCTURES
- AUTOMORPHISM GROUPS, ENDOMORPHISM MONOIDS, POLYMORPHISM CLONES
- RECONSTRUCTION NOTIONS
- THE GRAND TABLE OF RESULTS

PART II : METHODS

- PROPERTY X
- HOMOMORPHISM GLUING & HOMOGENEITY

MANY MATHEMATICAL OBJECTS CARRY: • ALGEBRAIC STRUCTURE • TOPOLOGICAL STRUCTURE } COMPATIBLE: ALGEBRAIC OPERATIONS CONTINUOUS.

EXAMPLES

- GROUP $(\mathbb{R},+)$ • GROUP $(\mathbb{R}^2,+)$
- VECTOR SPACES OVER $\mathbb{R}/\mathbb{C}/\mathbb{K}$: SUBSETS OF $\mathbb{R}^I / \mathbb{C}^I / \mathbb{K}^I$

  PRODUCT TOPOLOGIES

  IN PARTICULAR: $\mathbb{K}$ DISCRETE

  $\Rightarrow$ TOPOLOGY OF POINTWISE CONVERGENCE

  $(f_n)_{n\in\omega} \to f \iff \forall a \; f_n(a) = f(a)$ EVENTUALLY
- IN PARTICULAR: VECTOR SPACE $\mathbb{R}^n$
- GROUP $\mathrm{Aut}(\mathbb{Q},<) \subseteq \mathbb{Q}^{\mathbb{Q}}$: PRODUCT TOPOLOGIES
- IN GENERAL: GROUP $\mathrm{Aut}(\mathbb{A})$, $\mathbb{A}$ FIRST-ORDER STRUCTURE

  POINTWISE CONVERGENCE
- SEMIGROUPS (MONOIDS): $\mathrm{End}(\mathbb{A})$ ENDOMORPHISM MONOID

  $\mathrm{Emb}(\mathbb{A})$ EMBEDDING MONOID

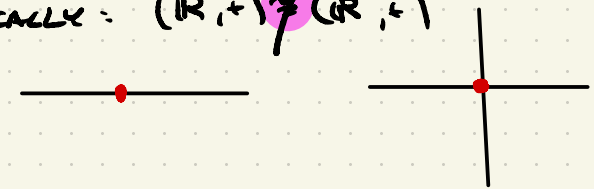
  $\mathrm{Aut}(\mathbb{A}) \subseteq \mathbb{A}^{\mathbb{A}}$
- CLONE $\mathrm{Pol}(\mathbb{A}) = \bigcup_{n\in\omega} \{f : \mathbb{A}^n \to \mathbb{A} \mid f \text{ PRESERVES } \mathbb{A}\}$

  ALGEBRAIC STRUCTURE: COMPOSITION

  $\mathrm{Pol}\,\mathbb{A} \cap \mathbb{A}^{\mathbb{A}^n}$ POINTWISE CONV., CLOPEN
- ALGEBRA $(A; (f_i)_{i\in I})$: COMPOSITION + POINTWISE CONVERGENCE.

HOW MUCH CHOICE FOR THE TOPOLOGIES?

- **UNIQUE**: VECTOR SPACE $\mathbb{R}^n$ HAS UNIQUE HAUSDORFF TOPOLOGY
- **MANY**: GROUPS $(\mathbb{R},+) \cong (\mathbb{R}^2,+)$ BUT NOT TOPOLOGICALLY: $(\mathbb{R},+) \not\cong (\mathbb{R}^2,+)$



QUESTION WHAT ARE THE COMPATIBLE TOPOLOGIES FOR A GIVEN ALGEBRAIC OBJECT?

QUESTION (MARKOV 44)

IS THERE AN INFINITE NON-TOPOLOGIZABLE GROUP?
(ONLY HAUSDORFF TOPOLOGY = DISCRETE)

ANSWER YES (SHELAH (CH), HESSE, OLSHANSKII '80)

QUESTION CAN THERE BE A UNIQUE NON-TRIVIAL, NON-DISCRETE TOPOLOGY?

- VARIANTS: ASSUMPTIONS ON TOPOLOGY, E.G., HAUSDORFF, POLISH

POLISH = SEPARABLE + COMPLETELY METRIZABLE

EXAMPLE POINTWISE CONVERGENCE ON $\mathrm{Sym}(\mathbb{N}) \subseteq \mathbb{N}^{\mathbb{N}} \subseteq \bigcup_{n\in\omega} \mathbb{N}^{\mathbb{N}^n}$ PW

POLISH GROUPS

- **EXAMPLE** $(\mathbb{R},+) \cong (\mathbb{R}^2,+)$, $(\mathbb{R},+) \not\cong (\mathbb{R}^2,+)$

- <u>ON THE OTHER HAND</u>: CONSISTENT WITH ZF$\cancel{C}$ THAT ANY POLISH GROUP HAS UNIQUE POLISH TOPOLOGY (PETTIS, SOLOVAY, SHELAH)

- **EXAMPLE** FREE GROUPS HAVE NO NON-DISCRETE POLISH TOPOLOGY (DUDLEY '61)

QUESTION (ULAM, SCOTTISH BOOK #96)

DOES $\mathrm{Sym}(\mathbb{N})$ HAVE LOCALLY COMPACT POLISH TOPOLOGY?

ANSWER <u>No</u>:

- EVERY T_1 TOPOLOGY $\geq$ PW (GAUGHAN '70s)
- $\mathcal{T} \subseteq \mathcal{T}'$, BOTH POLISH $\Rightarrow$ $\mathcal{T} = \mathcal{T}'$

$\Rightarrow$ PW UNIQUE POLISH

(EVEN SEPARABLE)

(ROSENDAL + SOLECKI '07)

UNIQUE POLISH TOPOLOGY:

- ISOMETRY GROUP OF THE URYSOHN SPACE / SPHERE (SABOK '13)
- HOMEOMORPHISM GROUPS OF $[0,1]^{\mathbb{N}}$, $2^{\mathbb{N}}$ (KALLMAN '80s)
- $\mathrm{Aut}(\mathbb{Q},<)$ (HODGES, HODKINSON, LASCAR, SHELAH '93 / SOLECKI + ROSENDAL '07)
- $\mathrm{Aut}(G)$ G ... RANDOM GRAPH (KRUSKOVSKI '92 / KECHRIS + ROSENDAL '07)

POLISH SEMIGROUPS / CLONES

UNIQUE POLISH TOPOLOGY:

- $\mathbb{N}^{\mathbb{N}}$
- $\bigcup_{n \in \omega} \mathbb{N}^{\mathbb{N}^n}$

(ELLIOTT + JONUŠAS + MESYAN + MITCHELL + MORAYNE + PÉRESSE '18)

INFINITELY MANY POLISH TOPOLOGIES:

$\mathrm{Inj}(\mathbb{N}) := \{f : \mathbb{N} \to \mathbb{N} \mid f \text{ INJECTIVE}\}$ $\left(= \overline{\mathrm{Aut}(\mathbb{N}, \neq)}^{pw}\right)$

SMALLEST: pw : GENERATED BY $U_{x,y} = \{f \in \mathbb{N}^{\mathbb{N}} \mid f(x) = y\}$ $(x, y \in \mathbb{N})$

LARGEST: pw + $N_x := \{f \in \mathbb{N}^{\mathbb{N}} \mid x \notin f(\mathbb{N})\}$ $(x \in \mathbb{N})$

$M_n := \{f \in \mathbb{N}^{\mathbb{N}} \mid |\mathbb{N} \setminus f(\mathbb{N})| = n\}$ $(n \leq \omega)$

IN BETWEEN: RESTRICT n

(NO CLASSIFICATION)

WHAT FOR?

INSIGHT INTO RELATIONSHIP BETWEEN ALGEBRAIC & TOPOLOGICAL STRUCTURE

PRACTICAL EVEN IF A TOPOLOGY IS "IRRELEVANT" (I.E. UNIQUELY DETERMINED), IT USUALLY IS FOR NON-TRIVIAL REASONS.
KNOWLEDGE OF THIS FACT IS USEFUL.

EXAMPLES LET $\mathbb{A}, \mathbb{B}$ BE COUNTABLE ω-CATEGORICAL: $\mathbb{A}^n/\mathrm{Aut}(\mathbb{A})$ FINITE

- $\mathbb{A}, \mathbb{B}$ FIRST-ORDER BI-INTERPRETABLE $\Leftrightarrow \mathrm{Aut}(\mathbb{A}) \cong \mathrm{Aut}(\mathbb{B})$ (AHLBRANDT + ZIEGLER '80s)
  IF $\mathbb{A}$ HAS UNIQUE POLISH TOP.: $\Leftrightarrow \mathrm{Aut}(\mathbb{A}) \cong \mathrm{Aut}(\mathbb{B})$
- $\mathbb{A}, \mathbb{B}$ EXISTENTIAL-POSITIVE BI-INT. $\Leftrightarrow \mathrm{End}(\mathbb{A}) \cong \mathrm{End}(\mathbb{B})$ (BODIRSKY + JUNKER '06)
- $\mathbb{A}, \mathbb{B}$ PRIMITIVE-POSITIVE BI-INT. $\Leftrightarrow \mathrm{Pol}(\mathbb{A}) \cong \mathrm{Pol}(\mathbb{B})$
  (BODIRSKY + P. '11)

VARIANT AUTOMATIC CONTINUITY AC

$\mathrm{Aut}(\mathbb{A})$ HAS AC $:\Leftrightarrow$ $\forall \varphi : \mathrm{Aut}(\mathbb{A}) \to \mathrm{Sym}(\mathbb{N})$ (φ CONTINUOUS)

$\mathrm{End}(\mathbb{A})$ HAS AC $:\Leftrightarrow$ $\mathrm{End}(\mathbb{A}) \to \mathbb{N}^{\mathbb{N}}$

$\mathrm{Pol}(\mathbb{A})$ HAS AC $:\Leftrightarrow$ $\mathrm{Pol}(\mathbb{A}) \to \bigcup_{n \in \mathbb{N}} \mathbb{N}^{\mathbb{N}^n}$

MEANING FOR ω-CATEGORICAL $\mathbb{A}$:

$\mathbb{B}$ HAS FO-INTERPRETATION IN $\mathbb{A}$ $\Leftrightarrow$ $\mathrm{Aut}(\mathbb{A}) \to \mathrm{Aut}(\mathbb{B})$

(MORE OR LESS)

EP- $\mathrm{End}(\mathbb{A}) \to \mathrm{End}(\mathbb{B})$

PP- $\mathrm{Pol}(\mathbb{A}) \to \mathrm{Pol}(\mathbb{B})$

EXAMPLE : CAN $\mathbb{A}$ PP-INTERPRET

$\mathbb{B} = \mathrm{SAT} := (\{0,1\}, \{(0,0,1), (0,1,0), (1,0,0)\})^2$:

$\mathrm{Pol}(\mathrm{SAT}) = \{\text{PROJECTIONS ON } \{0,1\}\}$

$\mathrm{Pol}(\mathbb{A}) \not\to \mathrm{Pol}(\mathrm{SAT}) \Rightarrow \mathrm{Pol}(\mathbb{A}) \models \mathrm{eos}(x,y,x,z,y,z) =$

$f \circ s(y,x,z,x,z,y)$

(BARTO + P. '16, NOT TRUE AS STATED)

RECONSTRUCTING THE TOPOLOGY OF $\mathrm{Aut}(\mathbb{A})$, $\mathrm{End}(\mathbb{A})$, $\mathrm{Pol}(\mathbb{A})$

$\mathbb{A}$ ... COUNTABLE ω-CATEGORICAL

THREE NOTIONS:

UP: PW IS UNIQUE POLISH TOPOLOGY ON $\underbrace{\mathrm{Aut}(\mathbb{A})}_{\text{HARD TO FAIL}} / \underbrace{\mathrm{End}(\mathbb{A}) / \mathrm{Pol}(\mathbb{A})}_{\text{FAILS EASILY}}$

AC: EVERY φ: $\mathrm{Aut}(\mathbb{A}) \to \mathrm{Sym}(\mathbb{N})$ } HARD TO FAIL IS CONTINUOUS

FAILS EASILY { $\mathrm{End}(\mathbb{A}) \to \mathbb{N}^{\mathbb{N}}$, $\mathrm{Pol}(\mathbb{A}) \to \bigcup_{n \in \omega} \mathbb{N}^{\mathbb{N}^n}$

(AUTOMATIC CONTINUITY)

AH: EVERY ISOMORPHISM φ: $\mathrm{Aut}(\mathbb{A}) \to \mathrm{Aut}(\mathbb{B})$, $\mathrm{End}(\mathbb{A}) \to \mathrm{End}(\mathbb{B})$, $\mathrm{Pol}(\mathbb{A}) \to \mathrm{Pol}(\mathbb{B})$ IS A HOMEOMORPHISM

HARD TO FAIL

EVANS HEWITT '90 + BODIRSKY & KOMPATSCHER + P. '15

(AUTOMATIC HOMEOMORPHICITY)

AR: EVERY ISOMORPHISM φ IS INDUCED BY BIJECTION $f: \mathbb{A} \to \mathbb{B}$

(ACTION RECONSTRUCTION)

$$\begin{tikzcd} & \text{AC} \arrow[dl, dashed] \\ \text{UP} \arrow[r] & \text{AH} \\ & \text{AR} \arrow[ul, dashed] \end{tikzcd}$$

FOR GROUPS (OPEN FOR MONOIDS)

AR USUALLY RESTRICTED TO $\mathbb{B}$ SATISFYING CONDITIONS: NO ALGEBRAIC

THE GRAND TABLE OF RESULTS

(INCOMPLETE + PROBABLY INCORRECT)

	UNIQUE POLISH (UP)	AUTOMATIC CONT. (AC)	ACTION REC. (AR)	AUTOMATIC HOMEO (AH)
Aut	$(\mathbb{N},=)$ Gaughan '70s $(\mathbb{Q},<)$ Rosendal + Solecki '07 $\mathcal{G}$ Kechris + Rosendal '07	$(\mathbb{N},=)$ Dixon, Neumann, Thomas '86, Simmons '81 $(\mathbb{Q},<)$, $\mathbb{B}$ Truss '89 $\mathcal{G}$, ω-stable Hodges + Hodkinson + Lascar + Shelah '80s $\mathbb{H}_n$ Henig '80s	$(\mathbb{Q},<)$, $\mathcal{G}$, $\mathbb{H}_n$, $\mathbb{P}$, $\mathbb{T}$ Rubin '94 Hypergraphs, Henson digraphs Bartoszynska, Macpherson '07	$(\mathbb{Q},<)$, $\mathcal{G}$, $\mathbb{H}_n$, $\mathbb{P}$, $\mathbb{T}$ Rubin '94
End	$\mathcal{G}$, $\mathbb{D}$, $\mathbb{E}$, $(\mathbb{P},<)$, $(\mathbb{P},\leq)$, $(\mathbb{Q},\leq)$ Elliott + Jonušas + Mitchell + Pérèse + P. + Schindler + '21 $(\mathbb{N},=)$ EJMMMP '20	$\mathcal{G}$, $\mathbb{D}$, $\mathbb{E}$ Elliott + Jonušas + Mitchell + Pérèse + P. '21 $(\mathbb{N},=)$ EJMMMP '20	$(\mathbb{Q},<)$, $(\mathbb{P},<)$, $\mathcal{G}$, $\mathbb{H}_n$ Bahrani + Vargas-García '19	$(\mathbb{N},+)$, $\mathcal{G}$, $\mathbb{T}$ Bodirsky + P. + Pongrácz '13 $(\mathbb{Q},<)$, $(\mathbb{Q},s)$ Bahrani + Truss + Vargas-García '17
Pol	$\mathcal{G}$, $\mathbb{D}$, $(\mathbb{P},<)$, $(\mathbb{P},\leq)$, $\mathbb{E}$ Elliott + Jonušas + Mitchell + Pérèse + P. '21 $(\mathbb{N},=)$ Elliott + Jonušas + Meysen + Mitchell + Morayne + Pérèse '20	$\mathcal{G}$, $\mathbb{D}$, $\mathbb{E}$ Elliott + Jonušas + Mitchell + Pérèse + P. '21 $(\mathbb{N},=)$ EJMMMP '20	$(\mathbb{Q},<)$, $(\mathbb{P},<)$, $\mathcal{G}$, $\mathbb{H}_n$ Bahrani + Vargas-García '19	$\geq \mathbb{N}^{\mathbb{N}}$, $\mathcal{G}$ Bodirsky + P. + Pongrácz '13 $(\mathbb{P},s)$, $(\mathbb{P},<)$ Peech + Peech '18 $(\mathbb{Q},\leq)$ Bahrani + Truss + Vargas-García '17 $(\mathbb{Q},<)$ Bahrani + Vargas-García '19

$\mathcal{G}$ ... RANDOM GRAPH

$\mathbb{P}$ ... RANDOM POSET

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ω-CAT

NO ALGEBRAICITY

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NEW METHOD PROPERTY $\mathbf{X}$

DEFINITION

S TOPOLOGICAL SEMIGROUP,
$A \subseteq S$ SUBSEMIGROUP.

S HAS PROPERTY $\mathbf{X}$ WITH RESPECT TO A
$:\Leftrightarrow$

$\forall s \in S \ \exists f_s, g_s \in S \ \exists \alpha_s \in A$:

$$s = g_s \circ \alpha_s \circ f_s$$

AND

$\forall B$ NEIGHBOURHOOD OF α_s :
$g_s \circ (B \cap A) \circ f_s$ IS NEIGHBOURHOOD OF s.

$= g_s \circ \alpha_s \circ f_s$

THEOREM (ELLIOTT + JONUŠAS + MEKSAH + MITCHELL MORAYNE + PÉRESSE '19)

LET S SEMIGROUP, POLISH TOPOLOGY $\mathcal{J}$ ON S.

$A \leq S$ SUBSEMIGROUP, PROPERTY $\mathbf{X}$.

- IF A IS POLISH SUBGROUP
  $\Rightarrow \mathcal{J}$ IS MAXIMAL POLISH TOPOLOGY ON S.
- A HAS AC
  $\Rightarrow S$ HAS AC.

COROLLARY

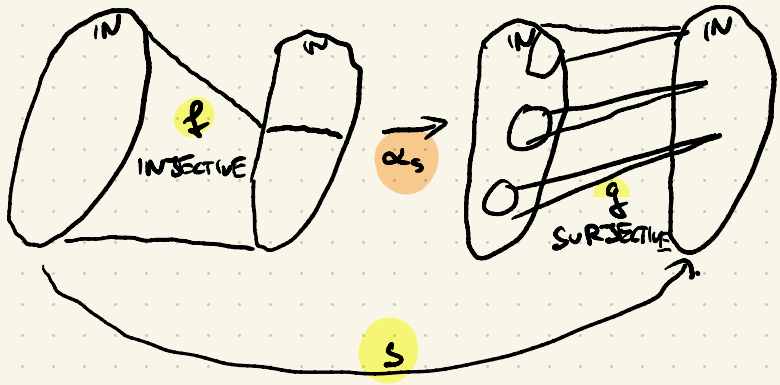
$\mathrm{End}(A)$ HAS PROPERTY $\mathbf{X}$ WRT $\mathrm{Aut}(A)$
$\Rightarrow$ PW MAXIMAL POLISH TOPOLOGY.

REMARK: OFTEN EASY TO SHOW THAT PW IS SMALLEST POLISH (EVEN HAUSDORFF) TOPOLOGY.

EXAMPLE $\mathbb{A} = (\mathbb{N}, =)$: $\mathrm{End}(\mathbb{A}) = \mathbb{N}^{\mathbb{N}}$, $\mathrm{Aut}(\mathbb{A}) = \mathrm{Sym}(\mathbb{N})$

PROPERTY X :

$s \in \mathbb{N}^{\mathbb{N}}$



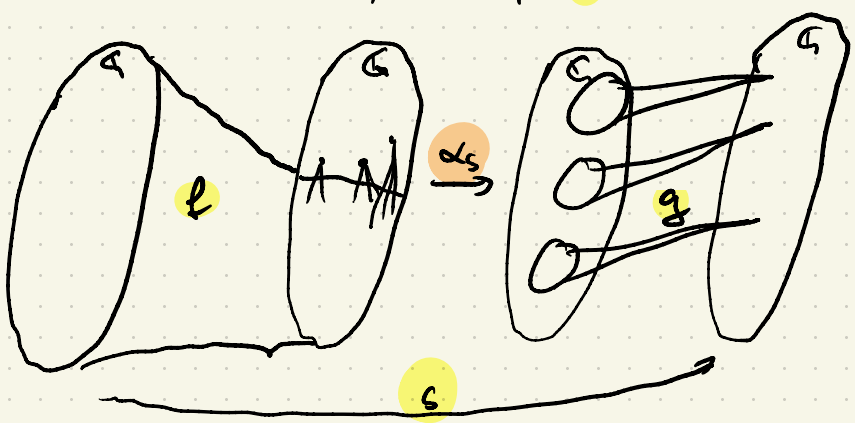
$s = g \circ \alpha_s \circ f$

SURJECTIVE INFINITE CLASSES

INJECTIVE CO-INFINITE RANGE

$\Rightarrow$ PW MAXIMAL POLISH TOP. ON $\mathbb{N}^{\mathbb{N}}$

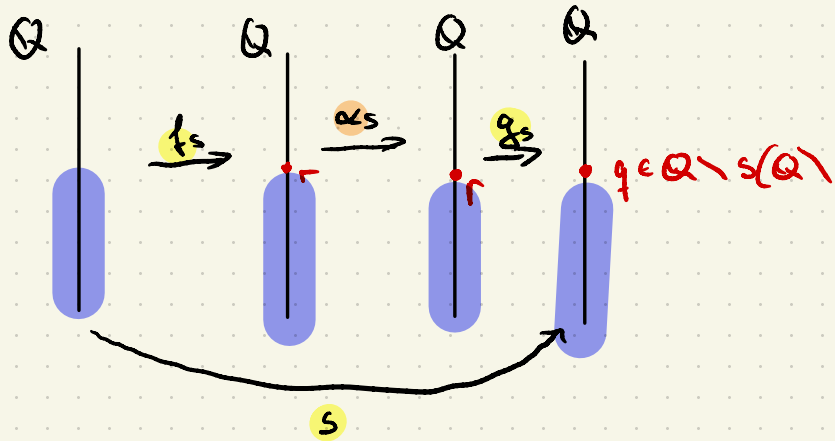
EXAMPLE $\mathbb{A} = G$ ... RANDOM GRAPH, $s \in \mathrm{End}\, G$



- g SURJECTIVE, "GENERIC"
- f EMBEDDING, GENERIC CO-IMAGE, PARAMETERS OF CO-IMAGE DO NOT DEFINE NEW TYPES IN THE IMAGE.
- α_s : BACK-AND-FORTH

NON-EXAMPLE

$\mathbb{A} = (Q, \leq)$



- $q \notin s(Q)$
- g_s HAS TO BE SURJECTIVE

$\Rightarrow \exists\, p \quad g_s(p) = q$

$\alpha_s(r) = p$

- ONCE $\alpha_s(r) = p$ IS FIXED, $g_s \circ \tilde{\alpha}_s \circ f_s$ MUST SEND BELOW q

- NO NEIGHBOURHOOD OF s IS THAT RESTRICTIVE!

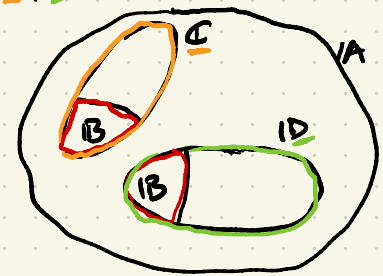
SUFFICIENT CONDITIONS FOR PROPERTY X

- $\mathbb{A}$ HOMOGENEOUS $:\Leftrightarrow$ EVERY PARTIAL FINITE ISOMORPHISM ON $\mathbb{A}$ EXTENDS TO AN AUTOMORPHISM.

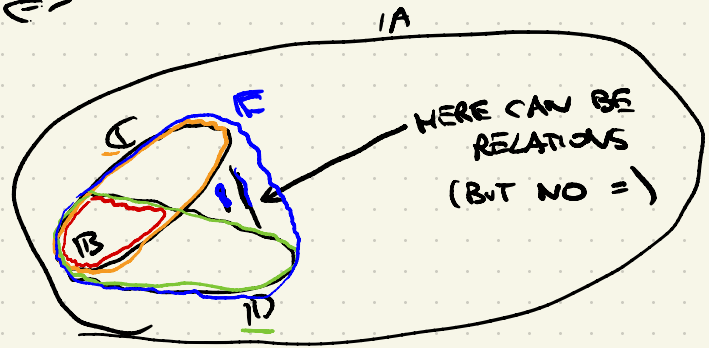
- $\mathbb{A}$ HOMOMORPHISM-HOMOGENEOUS $:\Leftrightarrow$ .... HOMOMORPHISM --- ENDOMORPHISM. (NOT $\triangle$-FREE GRAPH)

- $\mathbb{A}$ HAS STRONG AMALGAMATION PROPERTY $:\Leftrightarrow$

$\forall \mathbb{B}, \mathbb{C}, \mathbb{D}$ FINITE IN $\mathbb{A}$



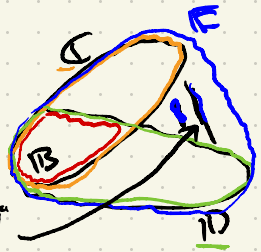
$\exists \mathbb{E}$ IN $\mathbb{A}$:



HERE CAN BE RELATIONS (BUT NO =)

SAP WITH HOMOMORPHISM GLUING:

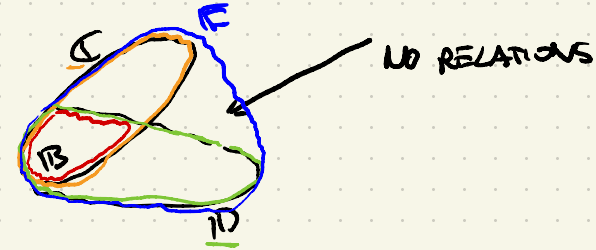
SAP :



$\oplus$ WHENEVER $g : \mathbb{E} \to \mathbb{F}$, $g|_{\mathbb{C}}$ HOMO, $g|_{\mathbb{D}}$ HOMO $\Rightarrow g$ HOMO

$\mathbb{E}$ IS "MINIMAL AMALGAM"

EXAMPLES



- FREE AMALGAMATION $\Rightarrow$ SAP + HC
- IN PARTICULAR: RANDOM GRAPH, DIGRAPH, ... HAVE SAP + HC
- RANDOM POSET $(\mathbb{P}, \leq)$, $(\mathbb{P}, <)$ HAVE SAP + HC
- EQUIVALENCE REL. $\mathbb{E}$ HAS SAP + HC
- $(\mathbb{Q}, \leq)$, $(\mathbb{Q}, <)$, $\mathbb{T}$ RANDOM TOURNAMENT: NO HC!

THEOREM (ELLIOTT + JONUŠAS + MITCHELL + PÉRESSE + P. '21)

LET $\mathbb{A}$ BE HOMOGENEOUS, HOMOMORPHISM-HOMOGENEOUS, SAP + HC.

$\Rightarrow$ $\mathrm{End}(\mathbb{A})$ HAS PROPERTY X WRT $\mathrm{Aut}(\mathbb{A})$,

PW IS THE MAXIMAL POLISH TOPOLOGY.

FAILS FOR $(\mathbb{Q}, \leq)$, HOWEVER:

THEOREM + C. SCHINDLER '21:

$\mathrm{End}(\mathbb{Q}, \leq)$ HAS UNIQUE POLISH TOPOLOGY

OPEN:

GENERAL ORDERED STRUCTURES, E.G. ORDERED RANDOM GRAPH!

Thank you!

THE GRAND TABLE OF RESULTS

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