

ALL MIXED IDENTITIES ARE SINGULAR
IN GROUPS WITHOUT ALGEBRAICITY

MICHAEL PINSKER
TU WIEN

MCW XXXI
PRAQUE

FWF Austrian
Science Fund

T5948



European Research Council
Established by the European Commission

SyG PococOP
GA 101071674

MY FIRST THEOREM IN ALGEBRA

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LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(\mathbb{Q}, <)$ SINGULAR?

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EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_r}_{\in \text{Aut}(Q, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$

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EXAMPLE: • $x_1 x_2 x_1^{-1} x_2^{-1} = 1$

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EXAMPLE:

- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ ✗
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- $[[y_1^x, y_3], [y_2^x, y_2]] = 1$

WHERE

$$\text{Supp } y_1 < \text{Supp } y_2 < \text{Supp } y_3$$

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„FORGETTING“
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W BECOMES
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E.G.

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CONNECTIONS WITH ZARISKI TOPOLOGY GIVEN BY SOLUTION SETS

TOPOLOGIES ON ENDOMORPHISM MONOIDS OF GENERIC STRUCTURES

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MCW XXIX
2024

$G = (G, \cdot, 1, {}^{-1})$ GROUP

REGULAR MIXED IDENTITIES 3

$G = (G, \cdot, 1, {}^{-1})$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_r) = 1$ HOLDS,

$u \in G, u \neq 1$

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CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

$G = \langle G, \cdot, 1, {}^{-1} \rangle$ GROUP

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THEOREM (ADELEKE, MOLLARD '94)

ALL $t(k_1 \dots x_n) = g$ SOLVABLE IN
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THEOREM (SCHNEIDER, THOM '22)

$\text{Sym}(N)$ NO REGULAR MIXED IDENTITIES

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THEOREM (GOLUBCHIK, MIKHAEV '82)

V VECTOR SPACE OVER INFINITE FIELD
 $\Rightarrow GL(V)$ NO MIXED IDENTITIES

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

$X \dots$ COUNTABLY INFINITE SET

$G \curvearrowright X$ OLIGOMORPHIC $\Leftrightarrow$

$\forall n \geq 1 \quad G \curvearrowright X^n$ FINITELY MANY ORBITS

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THEOREM AUTOMORPHISM GROUPS OF:

- RANDOM GRAPH ETENDJALABADI, CAO, LE MAÎTRE, MELERANT '21
- DIGRAPH DE LA NUEZ, GHADERNEZHAD '19
- POSET } BOBRSKY, THOM, SCHNIDER '24
- PERMUTATION }
- VECTOR SPACE OVER FINITE FIELD BRADFORD, THOM, SCHNEIDER
- HALL'S UNIVERSAL GROUP
- URYSON SPACE (ETENDJALABADI, CAO, LE MAÎTRE, MELERANT '21)

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$\text{Aut}(\mathbb{Q}, <) \text{ OPEN!}$

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"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{acl}(F) = F$

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EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓
- $\text{GL}(V)$ ✗

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STRONG AMALGAMATION (SAP)

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EXAMPLE

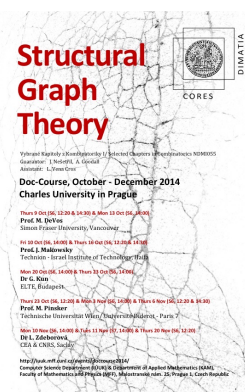
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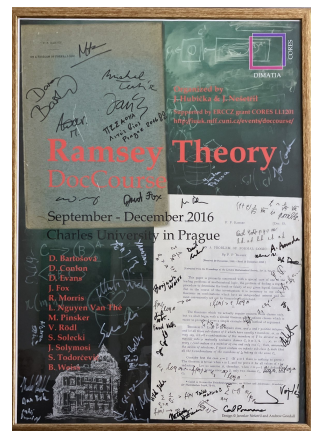
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HAPPY BIRTHDAY
JARIK !



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↳

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THEOREM (MARINON, P. '26)

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WHAT ABOUT VECTOR SPACES?

TRUE SCOPE OF THE THEOREM

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- $\forall F$ $\text{acl}(F)$ FINITE

- acl SATISFIES EXCHANGE:

$\forall x \forall y \forall F$

$x \in \text{acl}(F \cup \{y\}) \setminus \text{acl}(F)$

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"PREGEOMETRY"

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- acl MODULAR:

$$\begin{aligned} \dim(A \cup B) &= \dim(A) + \dim(B) \\ &\quad - \dim(A \cap B) \end{aligned}$$

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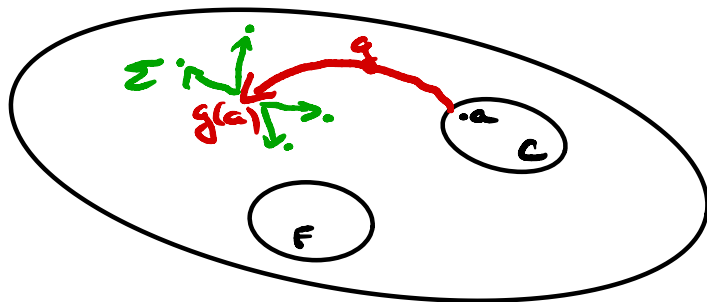
- acl HIGHER NEUMANN:

$$\forall F, C \quad \text{FINITE}$$

$$\forall a \notin \text{acl}(F)$$

$$\forall \Sigma \in G \quad \text{FINITE}$$

$$\exists g \in G_F \quad \Sigma \cdot g(a) \downarrow C$$



THEOREM (MARIMON, P. '26)

$G \curvearrowright X$, act : LOCALLY FINITE
EXCHANGE
MODULAR
HIGHER NEUMANN

$w(x_1, \dots, x_n, g_1, \dots, g_r) = 1$ REGULAR MIXED EQUATION

$\Rightarrow$ SOLUTION SET NOWHERE DENSE (POINTWISE)

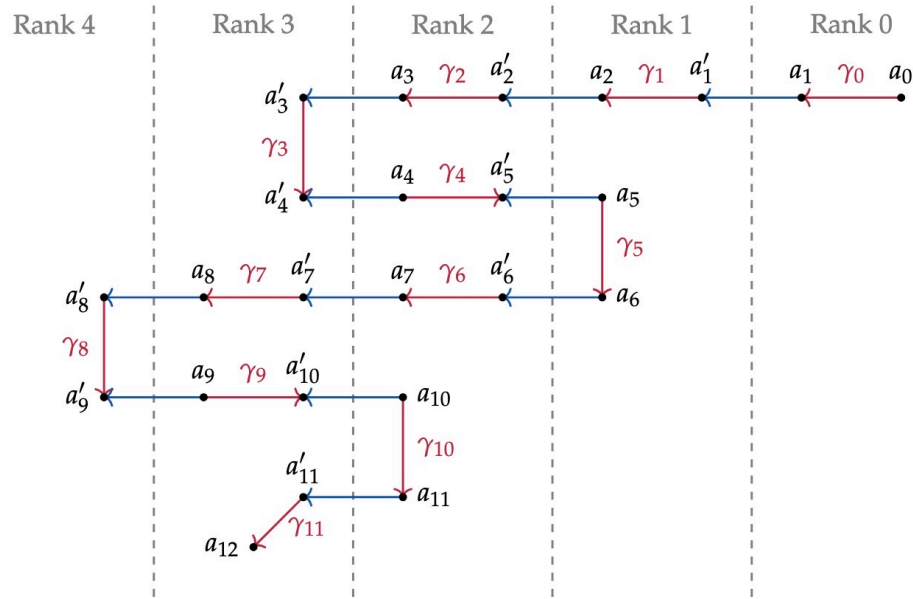
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"PINSKER'S LAST THEOREM"



$$y_{11} \times y_{10} \times^{-1} y_9 \times^{-1} y_8 \times y_7 \times y_6 \times y_5 \times^{-1} y_4 \times^{-1} y_3 \times y_2 \times y_1 \times y_0 = 1$$

Thank you!

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