DECIDABILITY OF INTERPRETABILITY

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TU WIEN

MATHS OF CSPs, DURHAM SYMPOSIUM, 2026

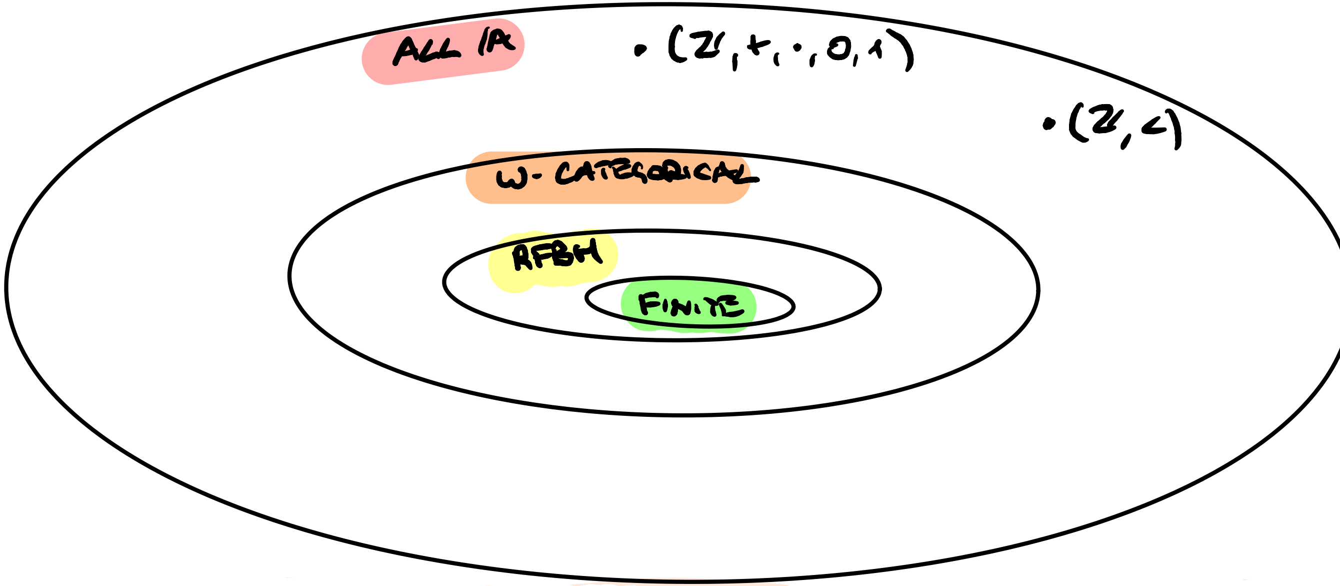
EUROPEAN RESEARCH COUNCIL

ERC SYNERGY GRANT POCOCOP (GA 101071674)

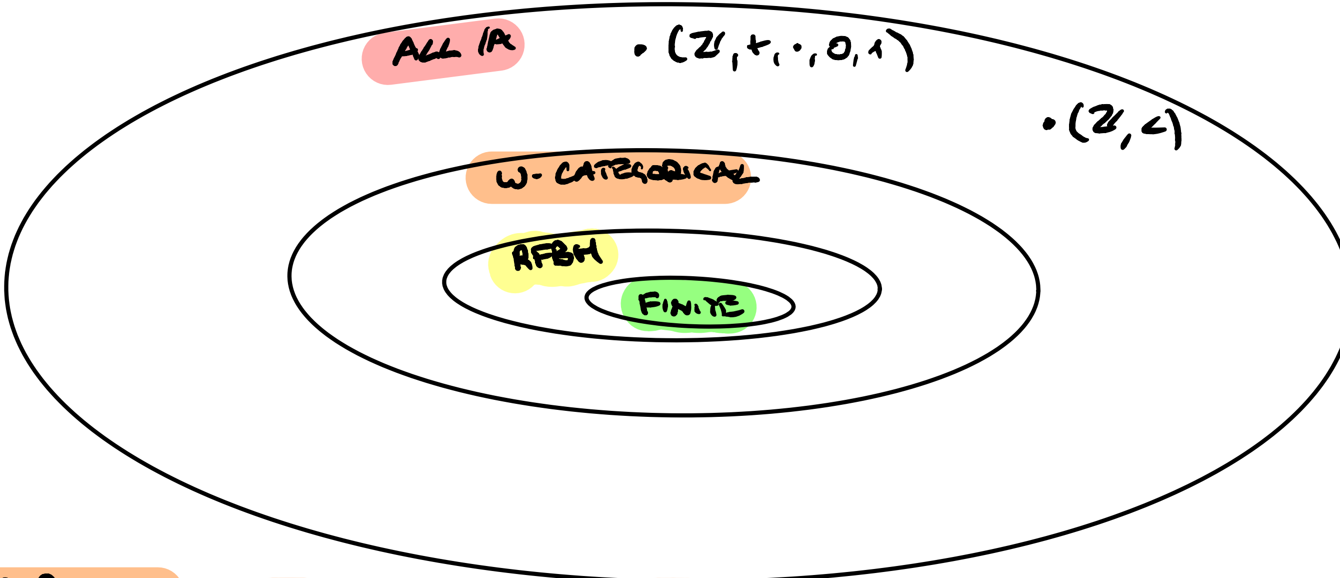
HO!

FWF I5948

$CSP(\mathbb{A})$


ALL $\mathbb{A}$
• $(\mathbb{Z}, +, \cdot, 0, 1)$
• $(\mathbb{Z}, <)$
ω- CATEGORICAL
RFBH
FINITE

CSP($\mathbb{A}$)



ω-CATEGORICAL: • $\mathrm{Aut}\,\mathbb{A} \curvearrowright A^n$ FINITELY MANY ORBITS $\forall n$

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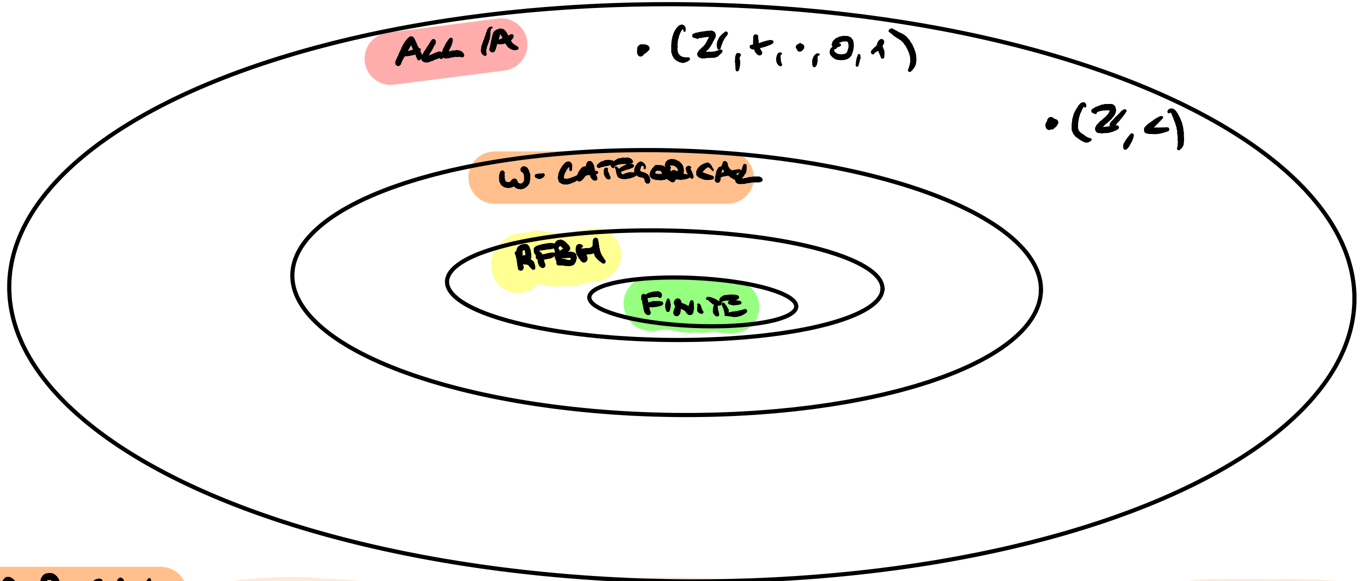

ω-CATEGORICAL:

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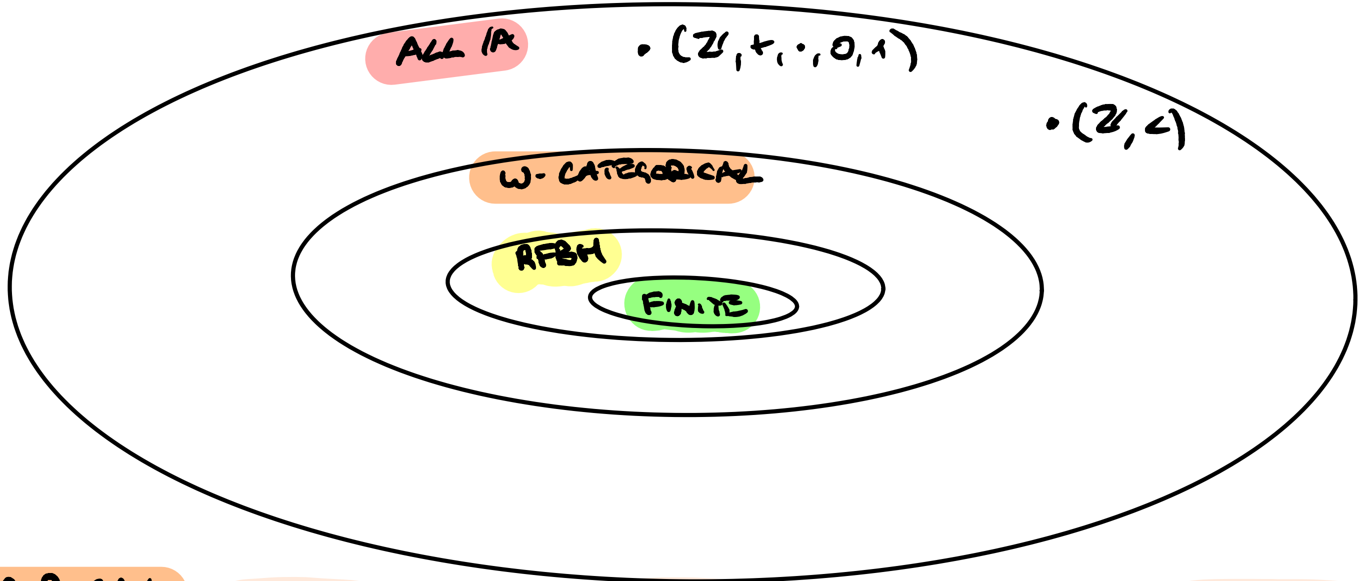

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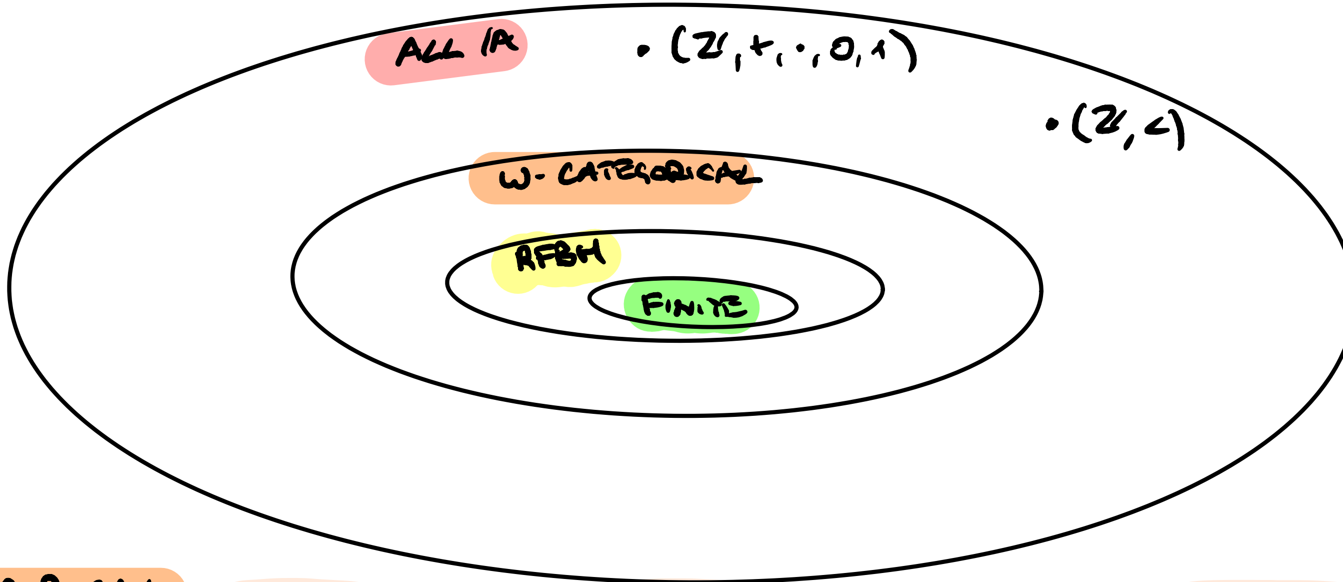
$\mathrm{CSP}(\mathbb{A})$



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RFBH:

(SEE C.S.)

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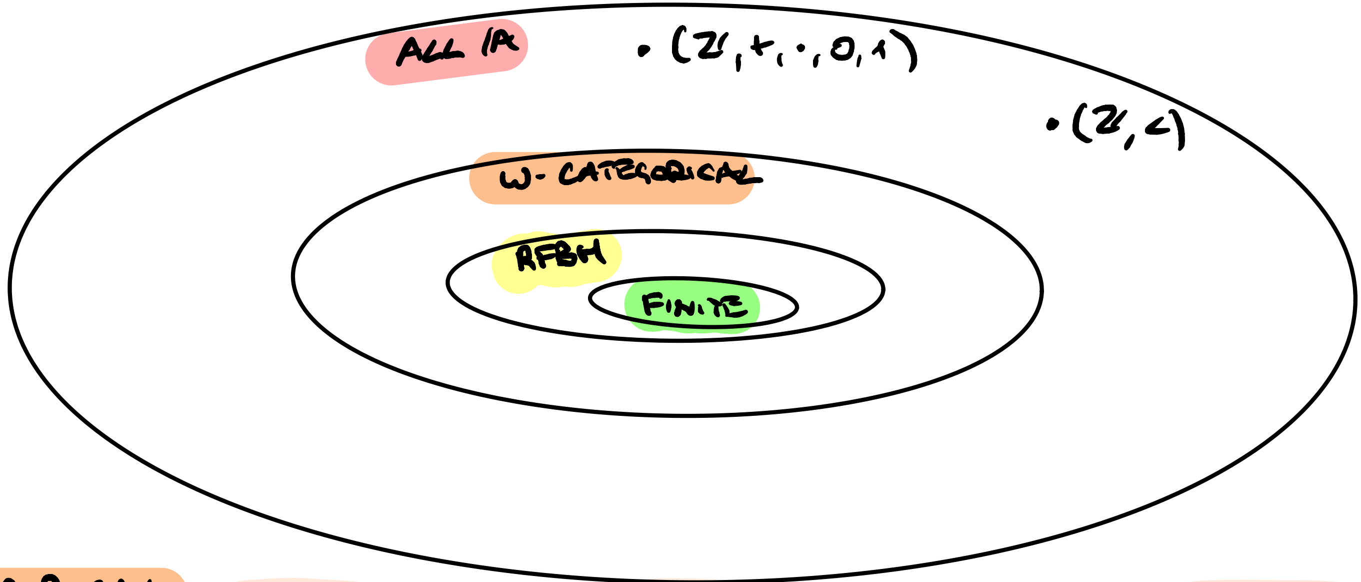

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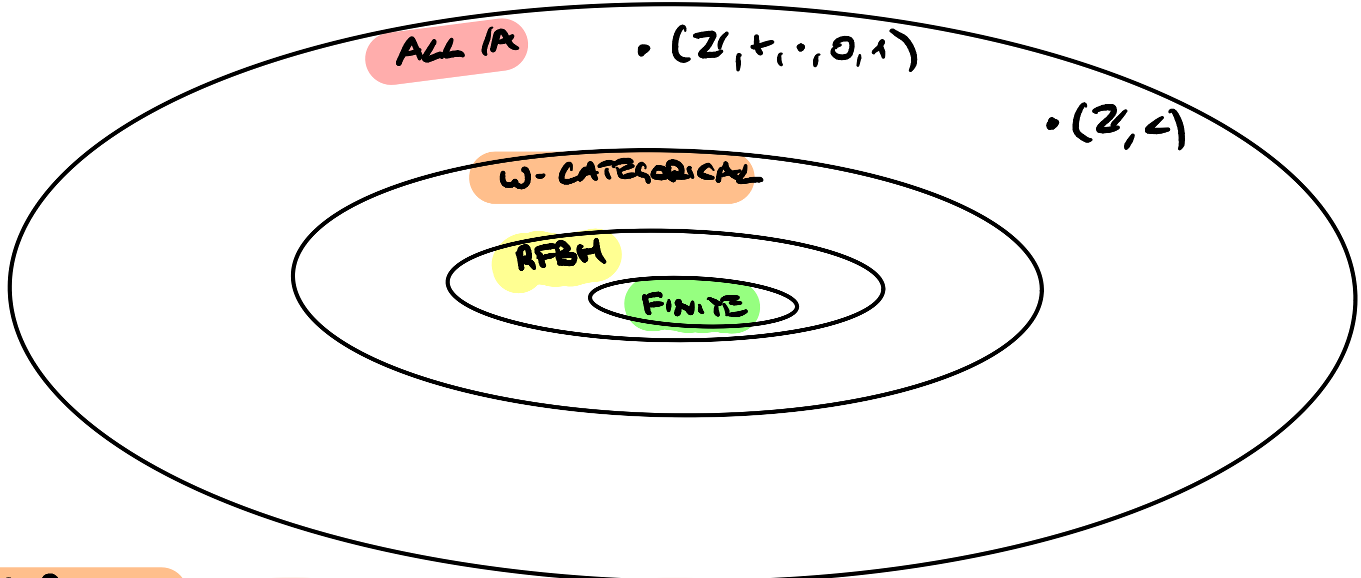
RFBH: (SEE C.S.)

- $\subseteq$ NP
- DICHOTOMY CONJECTURE

CSP($\mathbb{A}$)



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RFBH: (SEE C.S.)

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FINITE: DICHOTOMY

RFBH = REDUCT OF FINITELY BOUNDED HOMOGENEOUS

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EXAMPLE (SEE T.N.) (MMSNP)

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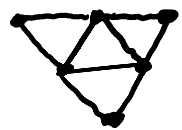

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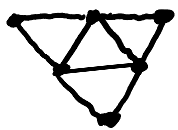
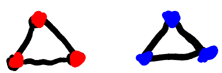


Q. COLOUR VERTICES WITH RED & BLUE AVOIDING



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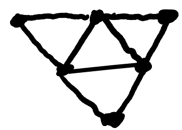

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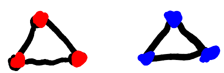

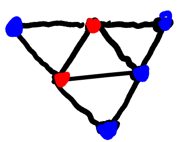

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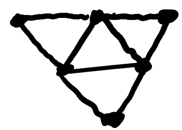

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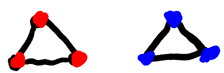

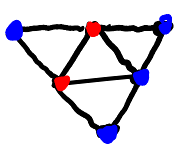

CSP(A) :

RFBH = REDUCT OF FINITELY BOUNDED HOMOGENEOUS

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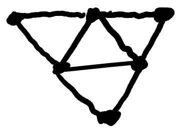

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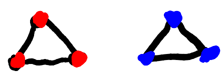
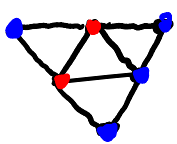
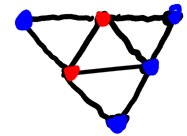
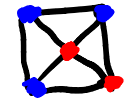




CSP(A) :

- TAKE ALL SOLUTIONS TO INSTANCES





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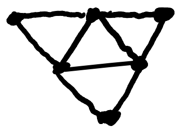

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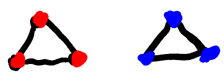

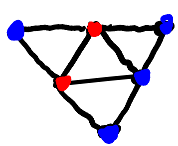

CSP($\mathbb{A}$) :

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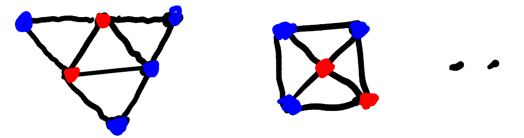
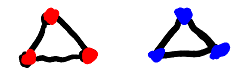


- GLUE THEM TOGETHER IN GENERIC WAY TO LIMIT $\mathbb{A}'$

$\mathbb{A}'$ IS FINITELY BOUNDED BY



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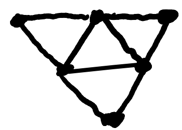

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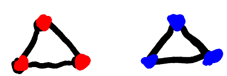

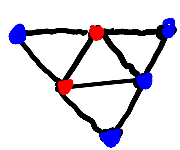

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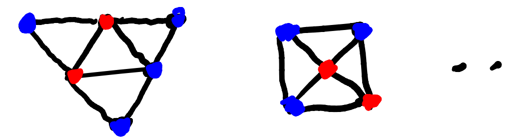

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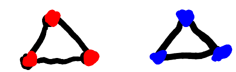

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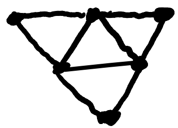

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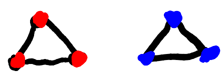

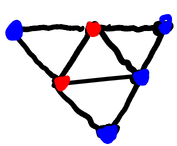

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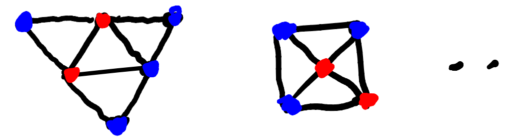

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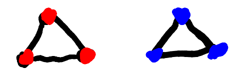

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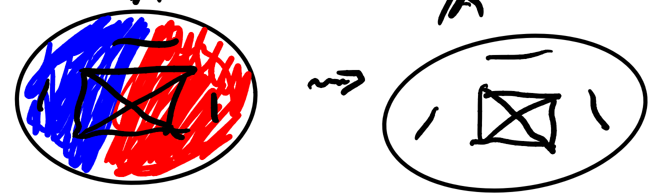
$\mathbb{A}'$ $\qquad$ $\mathbb{A}$

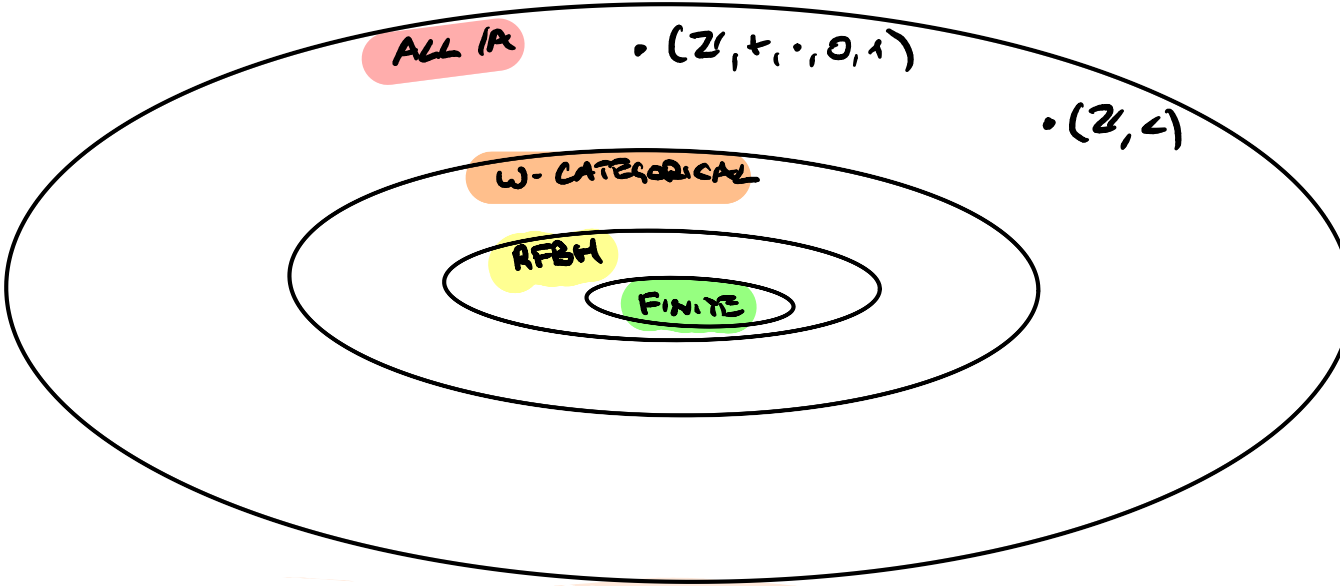


- $\mathbb{A}'$ HOMOGENEOUS!

ω-CATEGORICAL,
ORBIT OF $(t_1, \ldots, t_{1000000})$
DETERMINED BY PAIRS

(SEE M.S.)

CSP($\mathbb{A}$)



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ω- CATEGORICAL
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CLASSICS

CLASSICS

ω-CATEGORICAL

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ω-CATEGORICAL

- EVERY $\mathbb{A}$ HAS UNIQUE MC CORE $\mathbb{A}^c$:
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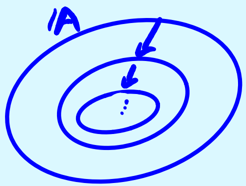

(BODIRSKY '07)

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  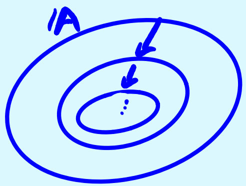

  (BODIRSKY '07)

- $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A}, \mathbb{B}$ PP-INTERDEF.

  (BODIRSKY + NEŠETŘIL '03)

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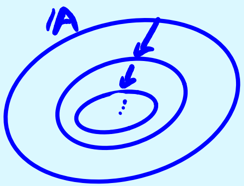

(BODIRSKY '04)

- $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A},\mathbb{B}$ PP-INTERDEF.
  (BODIRSKY + NEŠETŘIL '03)

- $\mathrm{Pol}\,\mathbb{A} \underset{\text{LOCALLY}}{\cong} \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A},\mathbb{B}$ PP-BI-INTERPR.
  (BODIRSKY + P. '15)

CLASSICS

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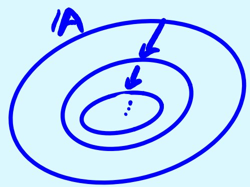

(BODIRSKY '07)

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  (BODIRSKY + NEŠETŘIL '06)

- $\mathrm{Pol}\,\mathbb{A} \underset{\text{LOCALLY}}{\cong} \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A}, \mathbb{B}$ PP-BI-INTERPR.
  (BODIRSKY + P. '15)

RFBH

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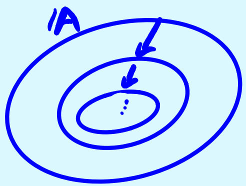

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  (BODIRSKY + NEŠETŘIL '03)

- $\mathrm{Pol}\,\mathbb{A} \underset{\text{LOCALLY}}{\cong} \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A}, \mathbb{B}$ PP-BI-INTERPR.

  (BODIRSKY + P. '15)

RFBH

- CORE RFBH IF $\mathbb{A}'$ RAMSEY (MOTTET + P. '2[illegible])

CLASSICS

ω-CATEGORICAL

- EVERY $\mathbb{A}$ HAS UNIQUE MC CORE $\mathbb{A}^c$:

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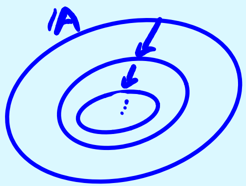

(BODIRSKY '07)

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  (BODIRSKY + NEŠETŘIL '03)

- $\mathrm{Pol}\,\mathbb{A} \underset{\text{LOCALLY}}{\cong} \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A}, \mathbb{B}$ PP-BI-INTERPR.

  (BODIRSKY + P. '15)

RFBH

- CORE RFBH IF $\mathbb{A}'$ RAMSEY (MOTTET + P. '20)

- $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B}$ DECIDABLE IF $\mathbb{A}' = \mathbb{B}'$ RAMSEY

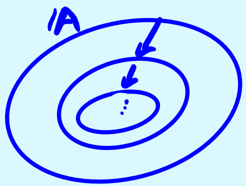
  (BODIRSKY + P. + TSANKOV '11)

  "DECIDABILITY OF DEFINABILITY"

CLASSICS

ω-CATEGORICAL

- Every $\mathbb{A}$ has unique MC core $\mathbb{A}^c$:
  CSP $\mathbb{A}$ = CSP $\mathbb{A}^c$, $\operatorname{Aut} \mathbb{A}^c \overset{\text{locally}}{=} \operatorname{End} \mathbb{A}^c$



(Bodirsky '07)

- $\operatorname{Pol} \mathbb{A} = \operatorname{Pol} \mathbb{B} \iff \mathbb{A}, \mathbb{B}$ pp-interdef.
  (Bodirsky + Nešetřil '03)
- $\operatorname{Pol} \mathbb{A} \overset{\text{locally}}{\cong} \operatorname{Pol} \mathbb{B} \iff \mathbb{A}, \mathbb{B}$ pp-bi-interpr.
  (Bodirsky + P. '15)

RFBH

- Core RFBH if $\mathbb{A}'$ Ramsey (Mottet + P. '20)
- $\operatorname{Pol} \mathbb{A} = \operatorname{Pol} \mathbb{B}$ decidable if $\mathbb{A}' = \mathbb{B}'$ Ramsey
  (Bodirsky + P. + Tsankov '11)
  "Decidability of definability"

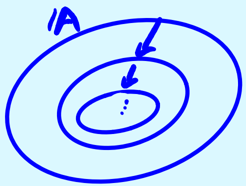
NEW

(Feller + P. '26)

CLASSICS

ω-CATEGORICAL

- EVERY $\mathbb{A}$ HAS UNIQUE MC CORE $\mathbb{A}^c$:
  CSP $\mathbb{A}$ = CSP $\mathbb{A}^c$, Aut $\mathbb{A}^c$ = End $\mathbb{A}^c$
  LOCALLY



(BODIRSKY '04)

- Pol $\mathbb{A}$ = Pol $\mathbb{B}$ $\Longleftrightarrow$ $\mathbb{A}, \mathbb{B}$ PP-INTERDEF.
  (BODIRSKY + NEŠETŘIL '03)
- Pol $\mathbb{A}$ $\cong$ Pol $\mathbb{B}$ $\Longleftrightarrow$ $\mathbb{A}, \mathbb{B}$ PP-BI-INTERPR.
  LOCALLY
  (BODIRSKY + P. '15)

RFBH

- CORE RFBH IF $\mathbb{A}'$ RAMSEY (MOTTET + P. '21)
- Pol $\mathbb{A}$ = Pol $\mathbb{B}$ DECIDABLE IF $\mathbb{A}' = \mathbb{B}'$ RAMSEY
  (BODIRSKY + P. + TSANKOV '11)
  "DECIDABILITY OF DEFINABILITY"

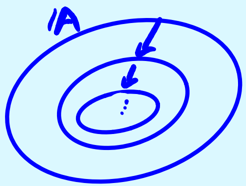
NEW (FELLER + P. '26)

ω-CATEGORICAL

CLASSICS

ω-CATEGORICAL

- EVERY $\mathbb{A}$ HAS UNIQUE MC CORE $\mathbb{A}^c$:
  CSP $\mathbb{A}$ = CSP $\mathbb{A}^c$, Aut $\mathbb{A}^c$ = End $\mathbb{A}^c$ LOCALLY



(BODIRSKY '07)

- Pol $\mathbb{A}$ = Pol $\mathbb{B}$ $\Longleftrightarrow$ $\mathbb{A}$, $\mathbb{B}$ PP-INTERDEF.
  (BODIRSKY + NEŠETŘIL '03)

- Pol $\mathbb{A}$ $\cong$ Pol $\mathbb{B}$ $\Longleftrightarrow$ $\mathbb{A}$, $\mathbb{B}$ PP-BI-INTERPR.
  LOCALLY
  (BODIRSKY + P. '15)

RFBH

- CORE RFBH IF $\mathbb{A}'$ RAMSEY (MOTTET + P. '21)

- Pol $\mathbb{A}$ = Pol $\mathbb{B}$ DECIDABLE IF $\mathbb{A}' = \mathbb{B}'$ RAMSEY
  (BODIRSKY + P. + TSANKOV '11)
  "DECIDABILITY OF DEFINABILITY"

NEW

(FELLER + P. '26)

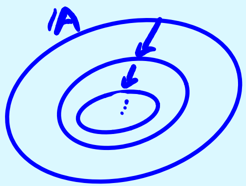
ω-CATEGORICAL

- Pol $\mathbb{A}$ $\cong$ Pol $\mathbb{B}$: SMOOTH EQUIVALENCE AMONG CORES WITHOUT ALGEBRAICITY
  TRANSITIVE

CLASSICS

ω-CATEGORICAL

- EVERY $\mathbb{A}$ HAS UNIQUE MC CORE $\mathbb{A}^c$:
  $CSP\,\mathbb{A} = CSP\,\mathbb{A}^c$, $Aut\,\mathbb{A}^c \underset{\uparrow \text{LOCALLY}}{=} End\,\mathbb{A}^c$



(BODIRSKY '04)

- $Pol\,\mathbb{A} = Pol\,\mathbb{B} \iff \mathbb{A},\mathbb{B}$ PP-INTERDEF.
  (BODIRSKY + NEŠETŘIL '03)
- $Pol\,\mathbb{A} \underset{\uparrow \text{LOCALLY}}{\cong} Pol\,\mathbb{B} \iff \mathbb{A},\mathbb{B}$ PP-BI-INTERPR.
  (BODIRSKY + P. '15)

RFBH

- CORE RFBH IF $\mathbb{A}'$ RAMSEY (MOTTET + P. '2[illegible])
- $Pol\,\mathbb{A} = Pol\,\mathbb{B}$ DECIDABLE IF $\mathbb{A}' = \mathbb{B}'$ RAMSEY
  (BODIRSKY + P. + TSANKOV '11)
  "DECIDABILITY OF DEFINABILITY"

NEW (FELLER + P. '26)

ω-CATEGORICAL

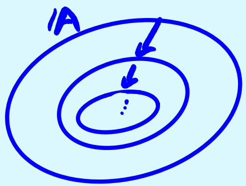
- $Pol\,\mathbb{A} \cong Pol\,\mathbb{B}$: SMOOTH EQUIVALENCE AMONG CORES WITHOUT ALGEBRAICITY TRANSITIVE

RFBH

CLASSICS

ω-CATEGORICAL

- EVERY $\mathbb{A}$ HAS UNIQUE MC CORE $\mathbb{A}^c$:
  $\mathrm{CSP}\,\mathbb{A} = \mathrm{CSP}\,\mathbb{A}^c$, $\mathrm{Aut}\,\mathbb{A}^c \underset{\text{LOCALLY}}{=} \mathrm{End}\,\mathbb{A}^c$



(BODIRSKY '07)

- $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A}, \mathbb{B}$ PP-INTERDEF.
  (BODIRSKY + NEŠETŘIL '03)
- $\mathrm{Pol}\,\mathbb{A} \underset{\text{LOCALLY}}{\cong} \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A}, \mathbb{B}$ PP-BI-INTERPR.
  (BODIRSKY + P. '15)

RFBH

- CORE RFBH IF $\mathbb{A}'$ RAMSEY (MOTTET + P. '20)
- $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B}$ DECIDABLE IF $\mathbb{A}' = \mathbb{B}'$ RAMSEY
  (BODIRSKY + P. + TSANKOV '11)
  "DECIDABILITY OF DEFINABILITY"

NEW (FELLER + P. '26)

ω-CATEGORICAL

- $\mathrm{Pol}\,\mathbb{A} \cong \mathrm{Pol}\,\mathbb{B}$: SMOOTH EQUIVALENCE AMONG CORES WITHOUT ALGEBRAICITY TRANSITIVE

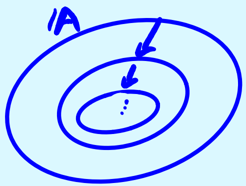
RFBH

- CORE IS COMPUTABLE: GIVEN $\mathbb{A}'$ FBH + RAMSEY, $\mathbb{A}$ REDUCT WE CAN OUTPUT $\mathbb{A}^c$ REDUCT OF $\mathbb{A}'$

CLASSICS

ω-CATEGORICAL

- Every $\mathbb{A}$ has unique MC core $\mathbb{A}^c$:
  $\mathrm{CSP}(\mathbb{A}) = \mathrm{CSP}(\mathbb{A}^c)$, $\mathrm{Aut}(\mathbb{A}^c) = \mathrm{End}(\mathbb{A}^c)$
  LOCALLY



(BODIRSKY '07)

- $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A}, \mathbb{B}$ PP-INTERDEF.
  (BODIRSKY + NEŠETŘIL '06)
- $\mathrm{Pol}\,\mathbb{A} \cong \mathrm{Pol}\,\mathbb{B} \iff \mathbb{A}, \mathbb{B}$ PP-BI-INTERPR.
  LOCALLY
  (BODIRSKY + P. '15)

RFBH

- CORE RFBH IF $\mathbb{A}'$ RAMSEY (MOTTET + P. '21)
- $\mathrm{Pol}\,\mathbb{A} = \mathrm{Pol}\,\mathbb{B}$ DECIDABLE IF $\mathbb{A}' = \mathbb{B}'$ RAMSEY
  (BODIRSKY + P. + TSANKOV '11)
  "DECIDABILITY OF DEFINABILITY"

NEW (FELLER + P. '26)

ω-CATEGORICAL

- $\mathrm{Pol}\,\mathbb{A} \cong \mathrm{Pol}\,\mathbb{B}$: SMOOTH EQUIVALENCE AMONG CORES WITHOUT ALGEBRAICITY
  TRANSITIVE

RFBH

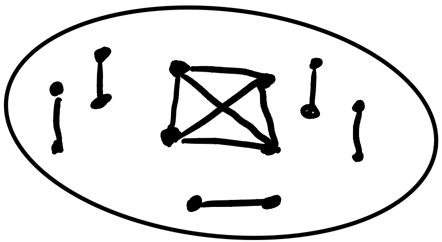
- CORE IS COMPUTABLE:
  GIVEN $\mathbb{A}'$ FBH + RAMSEY, $\mathbb{A}$ REDUCT
  WE CAN OUTPUT $\mathbb{A}^c$ REDUCT OF $\mathbb{A}'$"
- $\mathrm{Pol}\,\mathbb{A} \cong \mathrm{Pol}\,\mathbb{B}$: DECIDABLE

"DECIDABILITY OF INTERPRETABILITY"

COMPUTING THE MC CORE

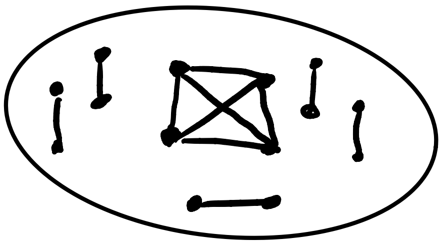
COMPUTING THE MC CORE

/A

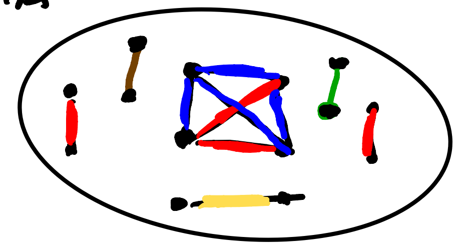


COMPUTING THE MC CORE

$\mathbb{A}$

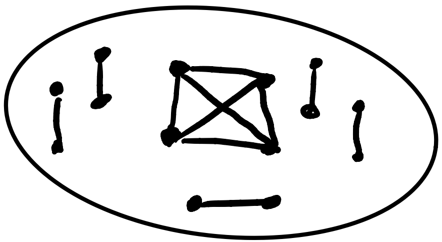


REDUCT OF FBH $\mathbb{A}'$



COMPUTING THE MC CORE

$\mathbb{A}$



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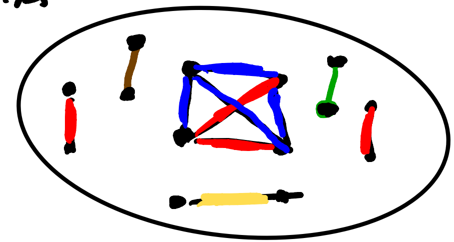

COMPUTING THE MC CORE

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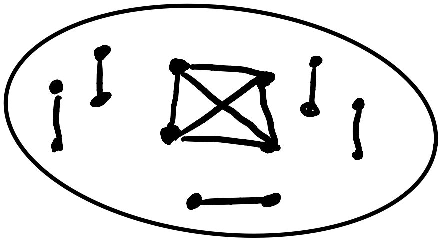

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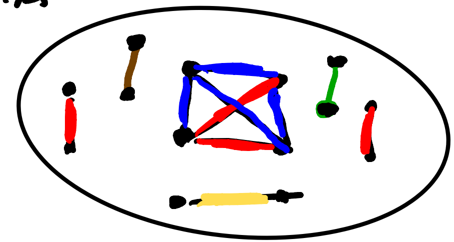
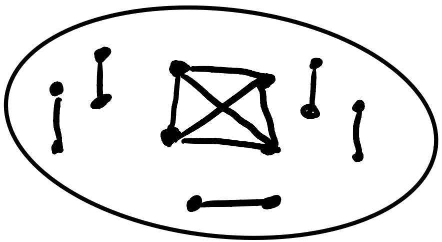


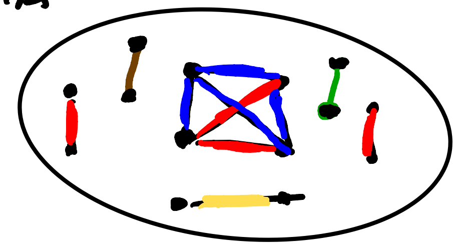
SHRINK $\mathbb{A}$ TO REMOVE ALL UNNECESSARY ORBITS OF $\mathbb{A}'$

COMPUTING THE MC CORE

$\mathbb{A}$



REDUCT OF FBH $\mathbb{A}'$



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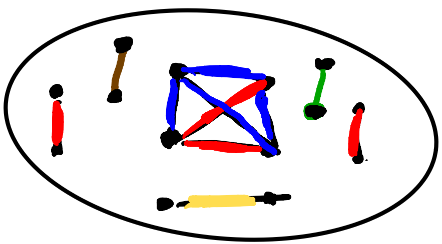
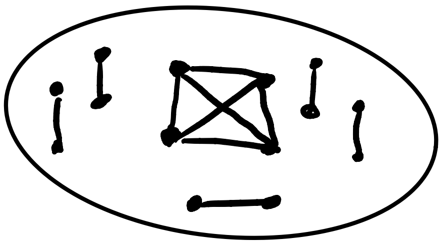


COMPUTING THE MC CORE

$\mathbb{A}$



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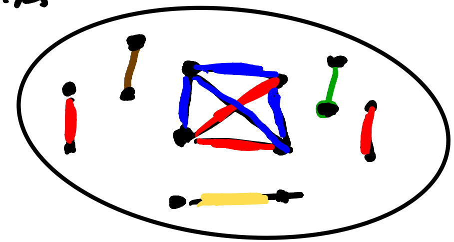

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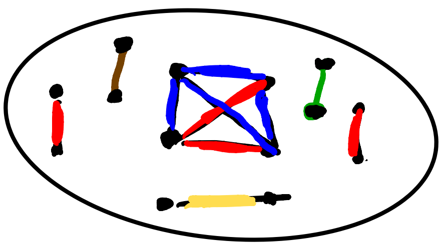
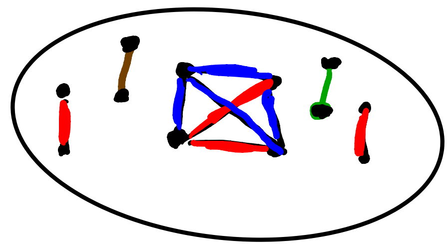
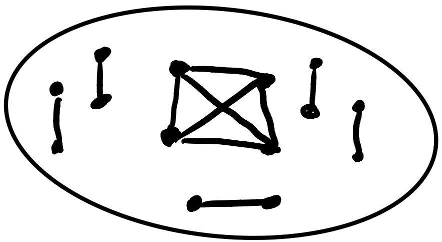


End $\rightsquigarrow$

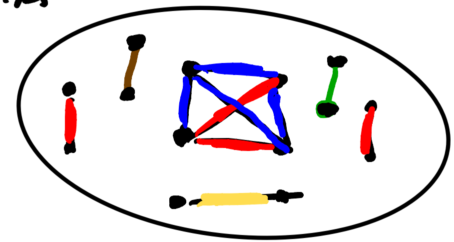


COMPUTING THE MC CORE

$\mathbb{A}$



REDUCT OF FBH $\mathbb{A}'$



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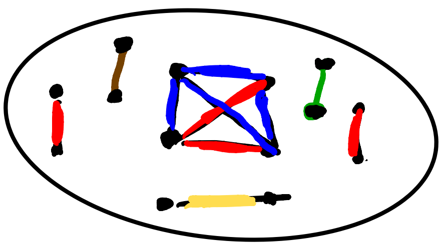

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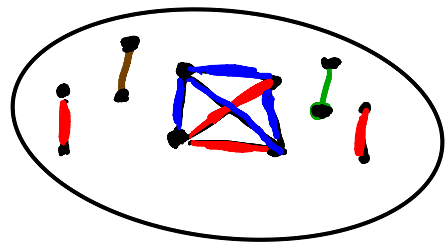

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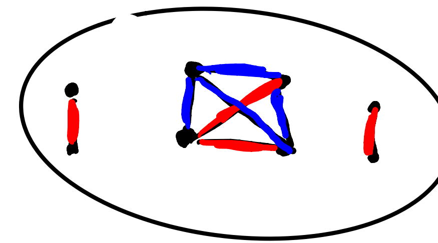
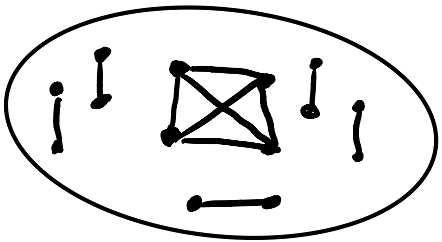
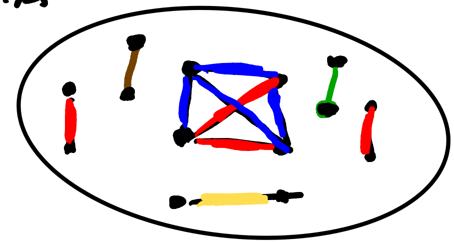


COMPUTING THE MC CORE

$\mathbb{A}$



REDUCT OF FBH $\mathbb{A}'$



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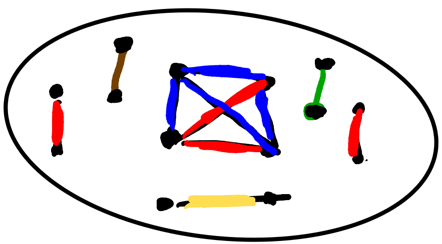
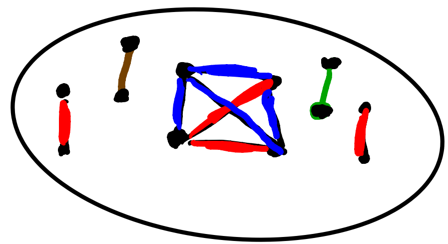
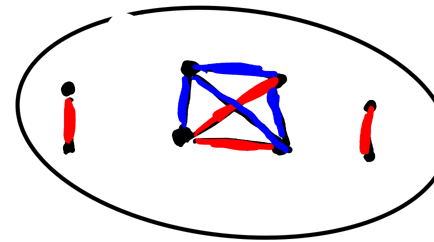


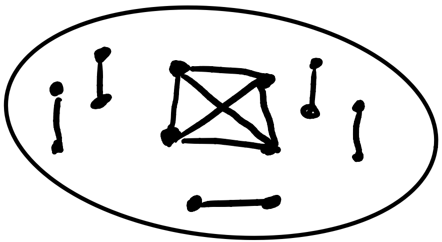




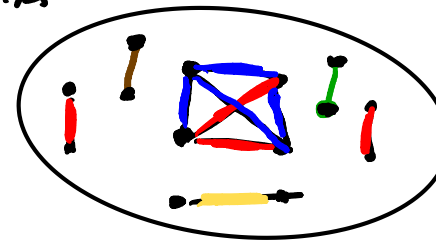
OBTAIN $\tilde{\mathbb{A}}$ RFBH $\tilde{\mathbb{A}}'$

COMPUTING THE MC CORE

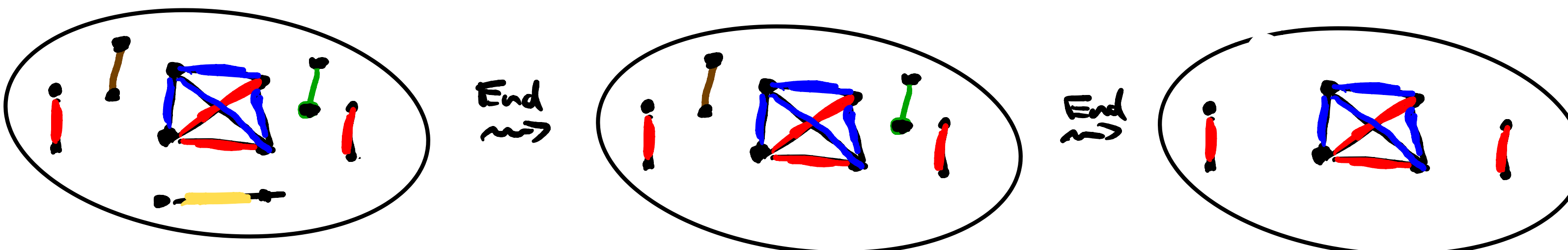
$\mathbb{A}$



REDUCT OF FBH $\mathbb{A}'$



SHRINK $\mathbb{A}$ TO REMOVE ALL UNNECESSARY ORBITS OF $\mathbb{A}'$



OBTAIN $\tilde{\mathbb{A}}$ RFBH $\tilde{\mathbb{A}}'$

THM (FELLER+P. '26)

$$\begin{array}{lcl} \text{SUPPOSE } \mathbb{A} \text{ RFBH } \mathbb{A}' & \rightsquigarrow & \tilde{\mathbb{A}} \text{ RFBH } \tilde{\mathbb{A}}' \\ h \downarrow \;\; \uparrow g & & \quad \cong \\ \mathbb{B} \text{ RFBH } \mathbb{B}' & \rightsquigarrow & \mathbb{B} \text{ RFBH } \tilde{\mathbb{B}}' \end{array}$$

FO BI-INT

SMOOTHNESS

SMOOTHNESS

X ... TOPOLOGICAL SPACE $\rightsquigarrow$ B_X BOREL SPACE:

σ-ALGEBRA GENERATED BY OPEN SETS

SMOOTHNESS

X ... TOPOLOGICAL SPACE $\rightsquigarrow$
B_X BOREL SPACE :
σ-ALGEBRA GENERATED BY OPEN SETS

DESCRIPTIVE SET THEORY :
$B_{\mathbb{R}} \cong B_{\{0\text{-}1\text{-}\mathrm{SEQUENCES}\}}$

SMOOTHNESS

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HIERARCHY:
Σ_1^1 ... PROJ. OF BOREL
Π_1^1 ... COMPLEMENTS
ETC.

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CLASSIFICATION PROBLEMS:
EQUIVALENCES ON $B_{\mathbb{R}}$

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CLASSIFICATION PROBLEMS:
EQUIVALENCES ON $B_{\mathbb{R}}$

E.G. "CLASSIFY ALL TORSION-FREE ABELIAN GROUPS ON $\mathbb{N}$"
(UP TO $\cong$)

SMOOTHNESS

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B_X BOREL SPACE:
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- E_1, E_2 EQUIVALENCE RELATIONS ON $\mathbb{R}$

$E_1 \leq_B E_2 :\Leftrightarrow \exists \varphi : \mathbb{R} \to \mathbb{R}$ BOREL
$\forall x, y \quad E_1(x, y) \Leftrightarrow E_2(\varphi x, \varphi y)$

SMOOTHNESS

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B_X BOREL SPACE:
σ-ALGEBRA GENERATED BY OPEN SETS

DESCRIPTIVE SET THEORY:
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Π^1_1 ... COMPLEMENTS
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- E SMOOTH $:\Leftrightarrow E \leq_B =_{\mathbb{R}}$

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(HARRINGTON + KECHRIS + LOUVEAU)

SMOOTHNESS

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Π^1_1 … COMPLEMENTS
ETC.

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(HARRINGTON + KECHRIS + LOUVEAU)

THM (PAOLINI + SHELAH '21)

$\cong$ ON TORSION-FREE ABELIAN GROUPS BOREL-COMPLETE

SMOOTHNESS

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HIERARCHY:

Σ_1^1 ... PROJ. OF BOREL

Π_1^1 ... COMPLEMENTS

ETC.

CLASSIFICATION PROBLEMS: EQUIVALENCES ON $B_{\mathbb{R}}$

E.G. "CLASSIFY ALL TORSION-FREE ABELIAN GROUPS ON $\mathbb{N}$" (UP TO $\cong$)

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(HARRINGTON + KECHRIS + LOUVEAU)

THM (PAOLINI + SHELAH '21)

$\cong$ ON TORSION-FREE ABELIAN GROUPS BOREL-COMPLETE

THM (NIES + PAOLINI '24)

FO-BI-INTERPRETABILITY SMOOTH ON ω-CAT STRUCTURES WITHOUT ALGEBRAICITY

THM (FELLER + P. '26)

PP-BI-INTERPRETABILITY SMOOTH ON

ω-CAT. CORES WITHOUT ALGEBRAICITY, TRANSITIVE

THM (FELLER + P. '26)

PP-BI-INTERPRETABILITY <u>SMOOTH</u> ON ω-CAT. CORES WITHOUT ALGEBRAICITY, TRANSITIVE

<u>PROOF</u> $\mathbb{A}, \mathbb{B}$ PP-BI-INT $\iff \mathrm{Pol}\,\mathbb{A} \cong \mathrm{Pol}\,\mathbb{B}$

$\iff \exists \varphi : A \to B \quad \mathrm{Pol}\,\mathbb{A} = \varphi^{-1}\,\mathrm{Pol}\,\mathbb{B}\,\varphi$

THM (FELLER + P. '26)

PP-BI-INTERPRETABILITY <u>SMOOTH</u> ON ω-CAT. CORES WITHOUT ALGEBRAICITY, TRANSITIVE

<u>PROOF</u> $\mathbb{A}, \mathbb{B}$ PP-BI-INT $\Leftrightarrow$ $\mathrm{Pol}\,\mathbb{A} \cong \mathrm{Pol}\,\mathbb{B}$

$\Leftrightarrow$ $\exists \varphi : A \to B$ $\mathrm{Pol}\,\mathbb{A} = \varphi^{-1}\, \mathrm{Pol}\,\mathbb{B}\, \varphi$

THM <u>DECIDABLE</u> ON RFBH RAMSEY

THM (FELLER + P. '26)

PP-BI-INTERPRETABILITY SMOOTH ON ω-CAT. CORES WITHOUT ALGEBRAICITY, TRANSITIVE

PROOF $\mathbb{A}, \mathbb{B}$ PP-BI-INT $\Leftrightarrow \operatorname{Pol} \mathbb{A} \cong \operatorname{Pol} \mathbb{B}$

$\Leftrightarrow \exists \varphi : A \to B \quad \operatorname{Pol} \mathbb{A} = \varphi^{-1} \operatorname{Pol} \mathbb{B}\, \varphi$

THM DECIDABLE ON RFBH RAMSEY

Thank you!

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