Abstract

Let s_q be the sum-of-digits function in base q, $q \geq 2$. If t is a positive integer, we denote by t^R the unique integer that is obtained from t by reversing the order of the digits of the proper representation of t in base q. In this work we prove that for all $\alpha \in \mathbb{R}$ and all positive integers t the correlation measure

$$\gamma(\alpha, t) = \lim_{x \to \infty} \frac{1}{x} \sum_{n<x} e^{2\pi i \alpha (s_q(n+t) - s_q(n))}$$

satisfies $\gamma(\alpha, t) = \gamma(\alpha, t^R)$. From this we deduce that for all integers d the sets $\{n \in \mathbb{N} : s_q(n+t) - s_q(n) = d\}$ and $\{n \in \mathbb{N} : s_q(n+t^R) - s_q(n) = d\}$ have the same asymptotic density. The proof involves methods coming from the study of q-additive functions, linear algebra, and analytic number theory.