ON A CONJECTURE OF CUSICK CONCERNING THE SUM OF DIGITS OF n AND $n+t$

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Abstract. For a nonnegative integer t, let c_t be the asymptotic density of natural numbers n for which $s(n+t) \geq s(n)$, where $s(n)$ denotes the sum of digits of n in base 2. We prove that $c_t > 1/2$ for t in a set of asymptotic density 1, thus giving a partial solution to a conjecture of T. W. Cusick stating that $c_t > 1/2$ for all t. Interestingly, this problem has several equivalent formulations, for example that the polynomial $X(X+1)\cdots(X+t-1)$ has less than 2^t zeros modulo 2^{t+1}. The proof of the main result is based on Chebyshev's inequality and the asymptotic analysis of a trivariate rational function using methods from analytic combinatorics.