Abstract

For a prime p and nonnegative integers j and n let $\vartheta_p(j,n)$ be the number of entries in the n-th row of Pascal's triangle that are exactly divisible by p^j. Moreover, for a finite sequence $w = w_{r-1} \cdots w_0 \neq 0 \cdots 0$ in $\{0,\ldots,p-1\}$ we denote by $|n|_w$ the number of times that w appears as a factor (contiguous subsequence) of the base-p expansion $n_{\mu-1} \cdots n_0$ of n. It follows from the work of Barat and Grabner (*Distribution of binomial coefficients and digital functions*, J. London Math. Soc. (2) 64(3), 2001), that $\vartheta_p(j,n)/\vartheta_p(0,n)$ is given by a polynomial P_j in the variables X_w, where w are certain finite words in $\{0,\ldots,p-1\}$, and each variable X_w is set to $|n|_w$. This was later made explicit by Rowland (*The number of nonzero binomial coefficients modulo p^α*, J. Comb. Number Theory 3(1), 2011), independently from Barat and Grabner's work, and Rowland described and implemented an algorithm computing these polynomials P_j. In this paper, we express the coefficients of P_j using generating functions, and we prove that these generating functions can be determined explicitly by means of a recurrence relation. Moreover, we prove that P_j is uniquely determined, and we note that the proof of our main theorem also provides a new proof of its existence. Besides providing insight into the structure of the polynomials P_j, our results allow us to compute them in a very efficient way.