

INTRODUCTION TO MATHEMATICAL LOGIC. WINTER SEMESTER 2014.

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Content of the course (= material required for the exam):

[3] without Chapter II, which will be replaced by [2, I.3-I.13], to be presented in the final part of the course. Deviations devoted to Model Theory are also planned. These will constitute a small part of [1]¹ and even less from [4].

The Exam will be oral.

Who wants to pass the exam immediately after the course can do it on Wednesday 28.01.2015 from 9:45 till 11.55 (there will be one more lecture on the same day, but it is planned as summarizing);
on Friday 30.01.2015 from 9:45 till 12:00;
on Monday 2.03.2015 from 10.00 till 12.00.

Please send me a short e-mail at least 2 days in advance!

Should you prefer to have an exam afterwards, any time which doesn't contradict the rules of the University is suitable for me. Again, an e-mail a couple of days in advance is needed!

Schedule.

Wednesday 15 : 15 – 16 : 50;

Friday 9 : 45 – 11 : 20.

First lecture: 08.10.2014.

Language: German, English on demand. It is also likely that I will switch to English once we reach Set Theory, unless some of you will object.

I will **assume** the familiarity of the material covered in “Grundbegriffe der mathematischen Logik”, see

<http://www.logic.univie.ac.at/~muellem3/teaching.html>

to have an idea which topics have been covered in this course last time. This material will be quickly repeated, though.

What have we already learned:

- *Lecture 1*, 08.10.2014 (Wed).
Chapter 1 of [3] until Lemma 2.2 (not yet proved).
- *Lecture 2*, 10.10.2014 (Fri).
Chapter 1 of [3] until Lemma 4.1.

¹Notation and terminology there is a bit different from that in [3], although it is rather natural. If you do not attend my lectures and have some troubles with understanding the notation or terminology in [1] feel free to ask me by e-mail.

- *Lecture 3*, 15.10.2014 (Wed.)
We reached the middle of the proof of the completeness theorem (made the first 2 steps from [3]).
- *Lecture 4*, 17.10.2014 (Fri.)
Mag. Hoffelner presented §5 of [3].
- *Lecture 5*, 22.10.2014 (Wed.)
The proof of the completeness theorem has been finished.
- *Lecture 6*, 24.10.2014 (Fri.)
Model theory. Covered the first 4 pages of [1].
- *Lecture 7*, 29.10.2014 (Wed.)
Reached [1, Bem. 2.3].
- *Lecture 8*, 31.10.2014 (Fri.)
Finished the first 3 paragraphs of [1]².
- *Lecture 9*, 05.11.2014 (Wed.)
Reached [1, Theorem 4.12].
- *Lecture 10*, 07.11.2014 (Fri.)
Finished Example 4.18(3) from [1].
- *Lecture 11*, 12.11.2014 (Wed.)
Reached [1, Theorem 4.26], i.e., we finished section 4 of [1].
- *Lecture 12*, 14.11.2014 (Fri.)
Section 8 from [4] till (incl.) Lemma 8.3.
- *Lecture 13*, 19.11.2014 (Wed.)
Started Chapter III of [3] and reached Lemma 13.6 there.
- *Lecture 14*, 21.11.2014 (Fri.)
Finished paragraph 14 of [3]. The lemma after 14.1 and Lemma 14.3 were not proved, but they are fairly easy modulo what we have done, and hence I expect you to read them by yourself.
- *Lecture 15*, 26.11.2014 (Wed.)
Reached Lemma 16.2 in [3].
- *Lecture 16*, 28.11.2014 (Fri.)
Finished with the proof of “Satz” right before the First Incompleteness Theorem (Folgerung 17.2), see [3]. Note that the “Folgerung” preceding this “Satz” is a simple corollary thereof.
- *Lecture 17*, 03.12.2014 (Wed.)
We reached Theorem 18.4 in [3].
- *Lecture 18*, 05.12.2014 (Fri.)
We finished by proving the Fixed Point Theorem on p. 95 of [3].
- *Lecture 19*, 10.12.2014 (Wed.)
We reached Theorem 19.1 in [3].
- *Lecture 20*, 12.12.2014 (Fri.)
We finished §19 except for the “Folgerung” in the middle of p. 102 of [3] We’ll not use it in the proofs of Gödel’s and Loeb’s theorems, so I will not present its proof, but I expect you to read its proof by yourselves.

²I will remove these files from my homepage after the course is finished.

- *Lecture 21*, 17.12.2014 (Wed.)
We finished the part of [3] I planned to present (Chapters I, III, IV) except for Lemma 20.1 and Theorem 20.3.
- *Lecture 22*, 07.01.2015 (Wed.)
We proved Lemma 20.1 and Theorem 20.3 from [3].
- *Lecture 23*, 09.01.2015 (Fri.)
We covered [2, §I.3 and §I.4].
- *Lecture 24*, 14.01.2015 (Wed.)
We reached Lemma I.6.16 in [2].
- *Lecture 25*, 16.01.2015 (Fri.)
We reached [2, Lemma I.7.4].
- *Lecture 26*, 21.01.2015 (Wed.)
We reached [2, Theorem I.7.21] and proved the last statement in it.
- *Lecture 27*, 23.01.2015 (Fri.)
We reached [2, Definition I.8.4].
- *Lecture 28*, 28.01.2015 (Wed.)
We reached [2, Theorem I.9.11] and proved it. This was the last lecture.

REFERENCES

- [1] Flum, J., *Modelltheorie*,
<http://www.logic.univie.ac.at/~lzdmsky/model1.pdf>
<http://www.logic.univie.ac.at/~lzdmsky/model2.pdf>
- [2] Kunen, K., *Set theory*. Studies in Logic (London), 34. College Publications, London, 2011.
- [3] Ziegler, M., *Mathematische Logik*, Reihe Mathematik Kompakt, Birkhäuser Verlag, 2010.
<http://home.mathematik.uni-freiburg.de/ziegler/skripte/logik.pdf>
- [4] Ziegler, M., *Vorlesung über Modelltheorie 1*,
<http://home.mathematik.uni-freiburg.de/ziegler/skripte/modell1.pdf>

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