AXIOMATIC SET THEORY 1. SUMMER SEMESTER 2012.

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The main aim of the course will be to present self-contained proofs of the facts that in L, the class of constructible sets, the following statements hold: Axiom of Choice (AC), Generalized Continuum Hypothesis, and $\diamondsuit$.

Content of the course (= material required for the exam) is a *proper* part of Chapters I and III-VI of the Kunen's book [1]. I.e., if you learn all these chapters you will know more than needed to pass the exam with the best note. During most of the lectures I will follow this book. My plan is to update this file every week. In particular, it will contain an up-to-date list of the topics which have already been covered. Should I decide to deviate from the Kunen's book, I will write some lecture notes.

Another book containing most of the information which I plan to present is [2].

The Exam will be oral. Who wants to pass the exam immediately after the course can do it
on Wednesday 20.06 from 8.30 till 13.00 (there will be one more lecture on the same day, but it will be summarizing, i.e., there will be hardly any new material presented there);
on Thursday 21.06 from 9.00 till 14.00;
on Friday 22.06 from 9.00 till 11.00 and from 19.30 till 21.00.

Please send me a short e-mail at least 2 days in advance!

Should you prefer to have an exam after June, any time which doesn't contradict the rules of the University is suitable for me. Again, an e-mail a couple of days in advance is needed!

Schedule.
Tuesday $12.20 - 13.50$;
Wednesday $13.00 - 13.45$.
First lecture: 06.03.2012.

Some parts of [1] are available at
http://www.logic.univie.ac.at/~lzdomsky/F.pdf

All necessary facts from mathematical logic we will use are available in
http://home.mathematik.uni-freiburg.de/ziegler/skripte/logik.pdf

Language: English.

What have we already learned

- Lecture 1, 06.03.2012 (Tue).
  Chapter 1 until the last paragraph on p. 13.

- Lecture 2, 07.03.2012 (Wed).
  Chapter 1 until (not including) Theorem 6.3.
- Lecture 3, 13.03.2012 (Tue).
  Chapter 1 until Definition 7.12.
- Lecture 4, 14.03.2012 (Wed).
  Chapter 1 until Definition 7.19.
- Lecture 5, 20.03.2012 (Tue).
  Chapter 1 until 9.4. Paragraph 8 was left out. You are welcome to read it yourself.
- Lecture 6, 21.03.2012 (Wed).
  Chapter 1 until 10.8.
- Lecture 7, 27.03.2012 (Tue).
  Chapter 1 until 10.27.
- Lecture 8, 28.03.2012 (Wed).
  Chapter 1 until the end of paragraph 10.
- Lecture 9, 17.04.2012 (Tue).
  Chapter 3 until Theorem 5.6.
- Lecture 10, 18.04.2012 (Wed).
  Chapter 3 until Definition 5.9.
- Lecture 11, 24.04.2012 (Tue).
  Chapter 4 until Lemma 2.9.
- Lecture 12, 25.04.2012 (Wed).
  Chapter 4 until Definition 3.5.
- Lecture 13, 02.05.2012 (Wed).
  Chapter 4 until Corollary 3.15. (not included).
- Lecture 14, 08.05.2012 (Tue).
  Chapter 4 until Lemma 5.8.
- Lecture 15, 09.05.2012 (Wed).
  Chapter 4 until Lemma 6.4
- Lecture 16, 15.05.2012 (Tue).
  Chapter 4 until Theorem 7.5.
- Lecture 17, 16.05.2012 (Wed).
  Chapter 4 until Theorem 7.8.
- Lecture 18, 22.05.2012 (Tue).
  Chapter 4 until the end of paragraph 7. Chapter 5 until Theorem 1.11. You are strongly encouraged to read the rest of Chapter 4.
- Lecture 19, 23.05.2012 (Wed).
  Chapter 5 until Definition 2.3
- Lecture 20, 30.05.2012 (Wed).
  Chapter 5 until the end.
- Lecture 21, 5.06.2012 (Tue).
  Chapter 6 until Theorem 3.5.
- Lecture 22, 6.06.2012 (Wed).
  Chapter 6 until Definition 4.1
- Lecture 23, 12.06.2012 (Tue).
  Chapter 6 until Lemma 4.11. Then we have introduced $\diamond$, proved some basic properties of clubs and the fact that $\diamond$ implies CH.

- Lecture 24, 13.06.2012 (Wed).
  Using the hint given in Exercise 13 after Chapter 6, we showed that $(V = L) \to \diamond$.
- Lecture 25, 19.06.2012 (Tue).
  We shall show that $\diamond$ implies the existence of a Souslin tree and derive some consequences of this fact.
- Lecture 26, 20.06.2012 (Wed).
  This will be the concluding lecture.

References

[1] Kunen, K., *Set theory. An introduction to independence proofs.* Studies in Logic and the Foundations of Mathematics, 102. North-Holland Publishing Co., Amsterdam-New York, 1980. xvi+313 pp.

[2] Jech, T., *Set theory.* The third millennium edition, revised and expanded. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003. xiv+769 pp.

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