READING COURSE IN MATHEMATICAL LOGIC.
WINTER SEMESTER 2016–2017.

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The main aim of the course will be to reach some interesting applications of forcing and large cardinals, e.g., in the set theory of reals.

Content of the course (= material required for the exam).
We will start from [1, §5] and try to reach famous Solovay's theorems about the measurability and the Baire property of sets of reals presented in §8 of this book.

The course will be self-contained modulo the material presented in Axiomatische Mengenlehre 1 taught in the previous semester by Vera Fischer, see `http://www.logic.univie.ac.at/~vfischer/SetTheory1.pdf`.

The Exam will be oral and include the material covered during the first 14 (i.e., all but the last one) lectures.

You can pass the exam on either of the following days:

1. 02.02.2017, 11:00-13:00.
2. 13.02.2017, 10:00-12:00.
3. Some day after Easter holidays, to be announced later.

Please send me a short e-mail at least 2 days in advance!

Should you prefer to have an exam on some other day, any time which doesn't contradict the rules of the University is suitable for me. Again, an e-mail a couple of days in advance is needed!

Schedule.
Monday, 12:20-13:50. First lecture: 10.10.2016

Language: English.

What have we already learned and reading material for the next lecture

- *Lecture 1, 10.10.2016*
  To be read: Pages 67–76 from [1][1].
  Done during the lecture: Until p.71 of [1].

- *Lecture 2, 17.10.2016*
  To be read: Pages 71–80 from [1].
  Done during the lecture: We've reached Definition 5.14.

[1] I have scans of some small parts of [1] thanks to my previous students. Please contact me if you need them.

- *Lecture 3, 24.10.2016*
  To be read: Until p. 83 from [1].
  Done during the lecture: We've introduced $<^E_\alpha$, see p. 79.

- *Lecture 4, 31.10.2016*
  To be read: Until §5.2 from [1].
  Done during the lecture: We've proven $\diamondsuit_\kappa(R)$ in $L[E]$ provided E has local condensation.

- *Lecture 5, 7.11.2016*
  To be read: §5 from [1] until the end.
  Done during the lecture: §5 from [1] until the end.

- *Lecture 6, 14.11.2016*
  To be read: §6 from [1] until Lemma 6.54[2].
  Done during the lecture: We've reached §6.2.

- *Lecture 7, 21.11.2016*
  To be read: §6 from [1] until the end.
  Done during the lecture: Until p. 114 of [1].

- *Lecture 8, 28.11.2016*
  To be read: Until p. 136 of [1].
  Done during the lecture: We've proved [1, Theorem 6.68] modulo one technical lemma, to be proved next time.

- *Lecture 9, 5.12.2016*
  To be read: Until p. 140 of [1].
  Done during the lecture: We've reached Lemma 7.8.

- *Lecture 10, 12.12.2016*
  To be read: §7 of [1] until the end.
  Done during the lecture: We've proven Lemma 7.12.

- *Lecture 11, 09.01.2017*
  To be read: Until the end of §8 of [1], at least superficially.
  Done during the lecture: We've reached the Shoenfield's absoluteness.

- *Lecture 12, 16.01.2017*
  To be read: Until [1, 8.11].
  Done during the lecture: We've reached Kondo's uniformization.

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[2]We will formulate and discuss all important facts from this paragraph, but the proof will be presented only of those of them that were not considered in the "Axiomatische Mengenlehre 1" course.

- *Lecture 13, 23.01.2017*
  To be read: Until the end of §8 of [1].
  Done during the lecture: Until the end of §7.

- *Lecture 14, 30.01.2017*
  To be read: Until the end of §8 of [1].
  Done during the lecture: We've reached the characterization of Cohen and random reals.

- *Lecture 15, 31.01.2017.* We've proven the Solovay's theorem stating that if κ is inaccessible, then in $V^{Col(\omega,<\kappa)}$ all $\mathrm{OD}_{\omega^\omega}$ sets of reals are Lebesgue measurable and have the Baire property.

References

[1] Schindler, R. *Set theory. Exploring independence and truth.* Universitext. Springer, Cham, 2014.

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