SPECIAL TOPICS IN SET THEORY. SOMMER SEMESTER 2022.

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The course will be devoted to applications of set theory to topology, this area of topology being known as *set theoretic topology*. In particular, in the course we shall consider some constructions of topological spaces using Souslin trees, diamonds, forcing (axioms), etc. Some of these will be complemented by impossibility results saying that these constructions cannot be carried out in ZFC. The course is going to be self-contained, modulo *basic* knowledge of general topology and *known* facts from set theory. Some of these set-theoretic facts will be also proven in the course, which exactly depending on the feedback from the audience during the course.

More information will appear here prior to the first lecture and as the course proceeds.

At the beginning of each lecture I will distribute a short script of the previous one. If you have missed some lectures and would like to have the script please send me an e-mail.

Schedule.
Friday 09:45-11:15. First lecture: 4.03.2022
Last lecture: 24.06.2018.

Exam.
30.06.2022, 30.09.2022, 25.11.2022, and 31.01.2023 at 12:00.
Please always send me an e-mail at least 3 days in advance in case you would like to come!
Other dates are also possible and can be arranged per e-mail.
Those who understand the most important relations between the central concepts in set theoretic topology to be addressed in the course (I believe attending the lectures would suffice for this) will surely get a positive note.

Language: English.

What we have done:

- *Lecture 1, 4.03.2022.* The preservation of separation axioms by finite products has been discussed.

- *Lecture 2, 25.03.2022.* The lecture was devoted to a characterization of Dowker spaces.

- *Lecture 3, 1.04.2022.* The main result presented in this lecture was the construction of a Souslin tree from $\diamondsuit$.

- *Lecture 4, 8.04.2022.* We started M.E. Rudin's construction from [11] of a Dowker space from a Souslin tree .

- *Lecture 5, 29.04.2022.* We have finished M.E. Rudin's construction of a Dowker space from a Souslin tree and discussed its basic properties.

- *Lecture 6, 06.05.2022.* We have started the construction of a Dowker space under CH from the second half of [4].

- *Lecture 7, 13.05.2022.* We finished the construction of a Dowker space under CH from the second half of [4].

- *Lecture 8, 20.05.2022.* We have checked that the space constructed in the previous two lectures under CH is indeed a Dowker space with many additional properties, e.g., it is an S-space.

- *Lecture 9, 27.05.2022.* We have discussed relations between CH, $\diamondsuit$, and $\clubsuit$.

- *Lecture 10, 03.06.2022.* We have presented a construction of an Ostaszewski space and related spaces under $\diamondsuit(=\mathrm{CH}+\clubsuit)$ and under $\clubsuit$, respectively, following [12, p. 35].

- *Lectures 11, 12 and 13. 10.06, 17.06. and 24.06.2022.* We have proved that *TOP* follows from *PFA*, following Baumgartner's chapter in [10].

References

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[12] Rudin, M.E., *Lectures on set theoretic topology,* Published for the Conference Board of the Mathematical Sciences by the American Mathematical Society, Providence, R.I., 1975. iv+76 pp.

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